Modeling in Information Retrieval
- Classical Models

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References:
1. *Modern Information Retrieval*, Chapter 3 & Teaching material
2. *Language Modeling for Information Retrieval*, Chapter 3

Modeling

- Produce a ranking function that assigns scores to documents with regard to a given query
  - Ranking is likely **the most important process** of an IR system
- This process consists of two main tasks
  - The conception of a logical framework for representing documents and queries
    - Sets, vectors, probability distributions, etc.
  - The definition of a ranking function (or retrieval model) that computes a rank (e.g., a real number) for each document in response to a given query

Index Terms

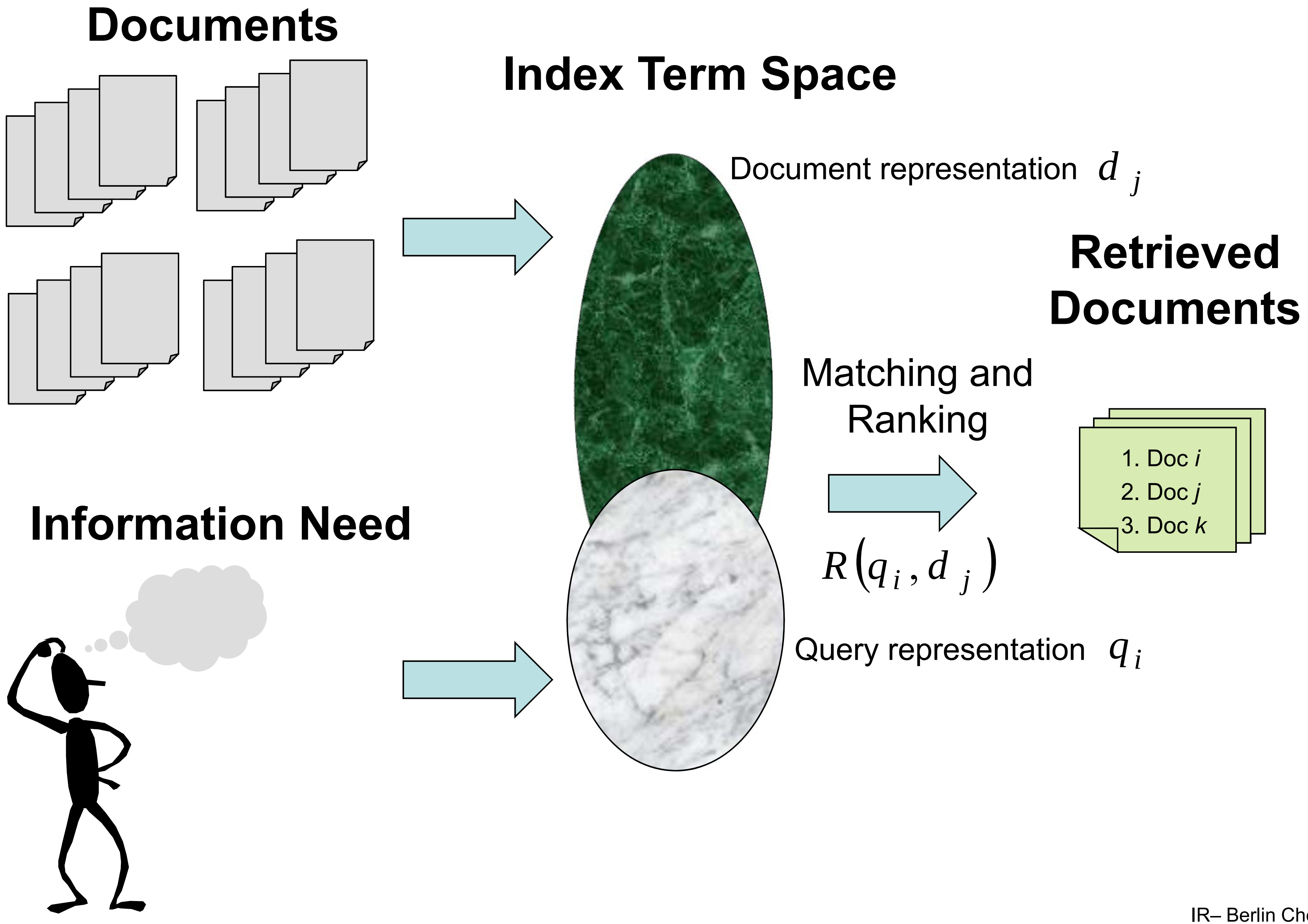
- Meanings From Two Perspectives
  - In a *restricted* sense (keyword-based)
    - An index term is a (predefined) keyword (usually a noun) which has some semantic meaning of its own
  - In a *more general* sense (word-based)
    - An index term is simply any word which appears in the text of a document in the collection
    - Full-text

Index Terms (cont.)

- The semantics (main themes) of the documents and of the user information need should be expressed through *sets of index terms*
  - Semantics is often lost when expressed through sets of words (e.g., *possible, probable, likely*)
    - Expressing query intent (information need) using a few words restricts the semantics of what can be expressed
  - Match between the documents and user queries is in the (*imprecise?*) space of index terms

Index Terms (cont.)

- Documents retrieved are flrequently irrelevant
  - Since most users have no training in query formation, problem is even worst
    - Not familar with the underlying IR process
    - E.g: frequent dissatisfaction of Web users
  - Issue of deciding document relevance, i.e. ranking, is critical for IR systems
    - A ranking algorithm predicts which documents the users will find relevant and which ones they will find irrelevant
      - Establish a simple ordering of the document retrieved; documents appearing on the top of this ordering are considered to be more likely to be relevant
    - However, two users might disagree what is relevant and what is not
      - Hopefully, the ranking algorithm can approximate the opinions of a large fraction of the users on the relevance of answers to a large fraction of queries


Documents
Index Term Space
Document representation d_j
Retrieved Documents
Matching and Ranking
1. Doc *i*
2. Doc *j*
3. Doc *k*
Information Need
$R(q_i, d_j)$
Query representation q_i

Ranking Algorithms

- Also called the "information retrieval models"

- Ranking Algorithms
  - Predict which documents are relevant and which are not
  - Attempt to establish a simple ordering of the document retrieved
  - Documents at the top of the ordering are more likely to be relevant
  - The core of information retrieval systems

Ranking Algorithms (cont.)

- A ranking is based on fundamental premises regarding the notion of document relevance, such as:
  - Common sets of index terms
  - Sharing of weighted terms
  - Likelihood of relevance

  literal-term matching

  $$P(Q|D) \text{ or } P(Q,D) \;?$$

  - Sharing of same aspects/concepts

  Concept/semantic matching

- Distinct sets of premises lead to a distinct IR models

Ranking Algorithms (cont.)

- Concept Matching vs. Literal Matching

Spoken Query

relevant ?

Transcript of Spoken Document

香港星島日報篇報導引述軍事觀察家的話表示，到二零零五年台灣將完全喪失空中優勢，原因是中國大陸戰機不論是數量或是性能上都將超越台灣，報導指出中國在大量引進俄羅斯先進武器的同時也得加快研發自製武器系統，目前西安飛機製造廠任職的改進型飛豹戰機即將部署尚未與蘇愷三十通道地對地攻擊住宅飛機，以督促遇到挫折的監控其戰機目前也已經取得了重大階段性的認知成果。根據日本媒體報導在台海戰爭隨時可能爆發情況之下北京方面的基本方針，使用高科技答應局部戰爭。因此，解放軍打算在二零零四年前又有包括蘇愷三十二期在內的兩百架蘇霍伊戰鬥機。

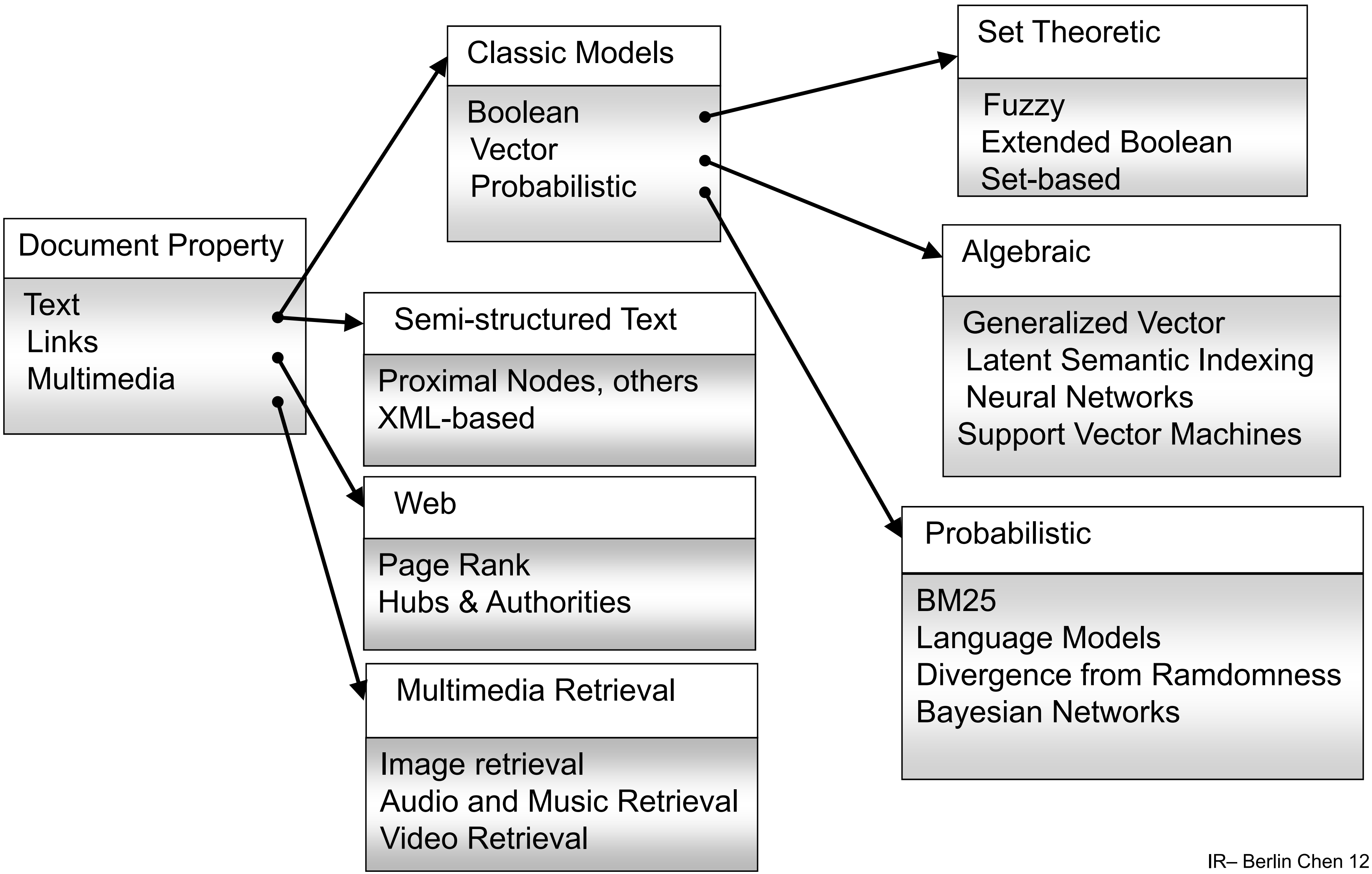
Taxonomy of Classic IR Models

- Refer to the text content
  - Unstructured
    - Boolean Model (*Set Theoretic*)
      - Documents and queries are represented as sets of index terms
    - Vector (Space) Model (*Algebraic*)
      - Documents and queries are represented as vectors in a *t*-dimensional space
    - Probabilistic Model (*Probabilistic*)
      - Document and query are represented based on probability theory
  - Semi-structured (Chapter 13)
    - Take into account the structure components of the text like titles, sections, subsections, paragraphs
    - Also include unstructured text

Taxonomy of Classic IR Models (cont.)

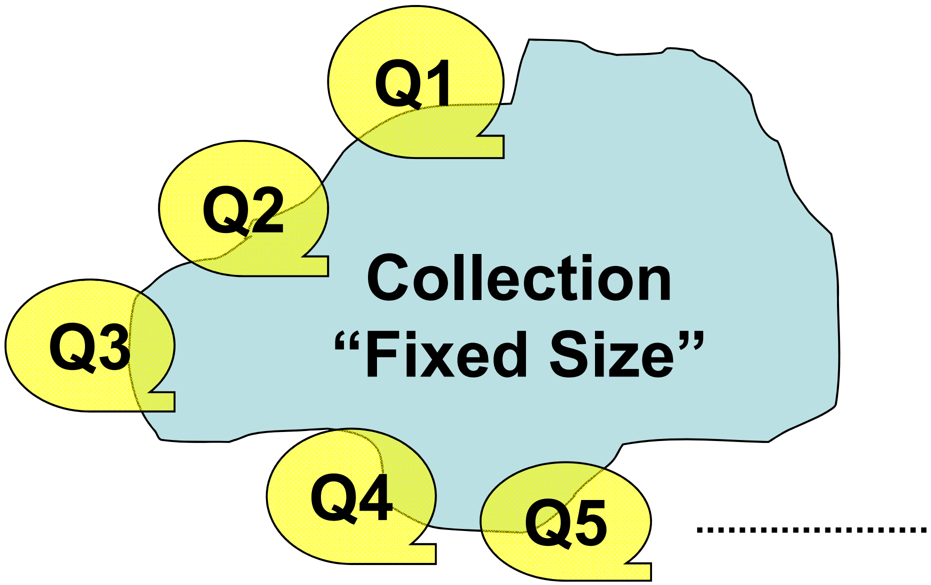
- Refer to the link structure of the Web (Chapter 11)
  - Consider the links among Web pages as an integral part of the model
- Refer to the content of multimedia objects (Chapter 14)
  - Images, video objects, audio objects

Taxonomy of Classic IR Models (cont.)



Retrieval: Ad Hoc

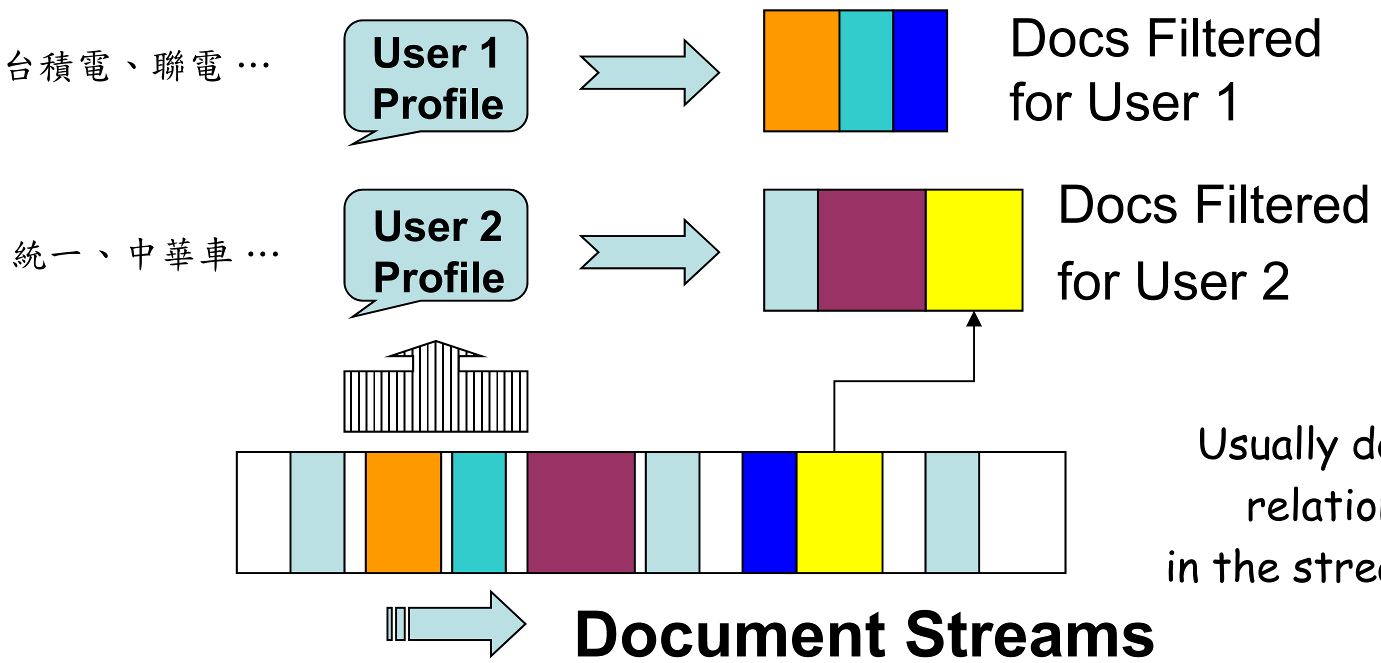
- *Ad hoc* retrieval
  - Documents remain relatively static while new queries are submitted to the system
    - The statistics for the entire document collection is obtainable
  - The most common form of user task



Ad hoc retrieval: Search based on a user query is sometimes called ad hoc search because the range of possible queries is huge and not pre-specified.

Retrieval: Filtering

- Filtering
  - Queries remain relatively static while new documents come into the system (and leave)
    - User profiles: Describe the users' preferences
  - E.g. news wiring services in the stock market



Usually do not consider the relations of documents in the streams (only **user task**)

Filtering & Routing

- **Filtering** task indicates to the user which document might be interested to him
    - Determine which ones are really relevant is fully reserved to the user
        - Documents with a ranking about a given threshold is selected
    - But no ranking information of filtered documents is presented to user
- **Routing**: a variation of filtering
    - Ranking information of the filtered documents is presented to the user
    - The user can examine the Top *N* documents
- The vector model is preferred (for simplicity!)
    - **For filtering/routing, the crucial step is not ranking but the construction of user profiles**

Filtering: User Profile Construction

- Simplistic approach
  - Describe the profile through a set of keywords
  - The user provides the necessary keywords
  - User is not involved too much
  - Drawback: If user not familiar with the service (e.g. the vocabulary of upcoming documents)

- Elaborate approach
  - Collect information from user the about his preferences
  - Initial (primitive) profile description is adjusted by relevance feedback (from relevant/irrelevant information)
    - User intervention
  - Profile is continuously changing

A Formal Characterization of IR Models

- The quadruple /**D**, **Q**, *F*, $R(q_i,d_j)$/ definition
  - **D**: a set composed of logical views (or representations) for the documents in collection
  - **Q**: a set composed of logical views (or representations) for the user information needs, i.e., "queries"
  - *F*: a framework for modeling documents representations, queries, and their relationships and operations
  - $R(q_i, d_j)$: a ranking function which associates a real number with $q_i \in \mathbf{Q}$ and $d_j \in \mathbf{D}$
    - Define an ordering among the documents d_j with regard to the query q_i

A Formal Characterization of IR Models (cont.)

- Classic Boolean model
  - Set of documents
  - Standard operations on sets
- Classic vector model
  - *t*-dimensional vector space
  - Standard linear algebra operations on vectors
- Classic probabilistic model
  - Sets (relevant/irrelevant document sets)
  - Standard probabilistic operations
    - Mainly the Bayes' theorem

Basic Concepts

- Each document represented by a set of representative keywords or index terms
- An index term is a **word** or **group of consecutive words** in a document whose semantics is useful for remembering (summarizing) the *document main themes*
- Usually, index terms are *nouns* because nouns have meaning by themselves
  - Adjectives, adverbs, and connectives mainly work as complements
- However, search engines assume that all words are index terms (full text representation)

Basic Concepts (cont.)

- Let,
  - t be the number of index terms in the document collection
  - k_i be a generic index term
- Then,
  - The **vocabulary** $V = \{k_1, \ldots, k_t\}$ is the set of all distinct index terms in the collection

$$V = \boxed{k_1 \quad k_2 \quad k_3 \quad \cdots \quad k_t}$$

vocabulary of t index terms

Basic Concepts (cont.)

- Documents and queries can be represented by **patterns of term co-occurrences**

$$V = \boxed{k_1 \quad k_2 \quad k_3 \quad \ldots \quad k_t}$$

$$\boxed{1 \quad 0 \quad 0 \quad \ldots \quad 0}$$ pattern that represents documents (and queries) with the term k_1 and no other

$$\vdots$$

$$\boxed{1 \quad 1 \quad 1 \quad \ldots \quad 1}$$ pattern that represents documents (and queries) with all index terms

- Each of these patterns of term co-occurrence is called a **term conjunctive component**
- For each document d_j (or query q) we associate a unique term conjunctive component $c(d_j)$ (or $c(q)$)

The Term-Document Matrix

- The occurrence of a term k_i in a document d_j establishes a relation between k_i and d_j
- A **term-document relation** between k_i and d_j can be quantified by the frequency of the term in the document
- In matrix form, this can written as

$$
\begin{array}{c c}
 & \begin{array}{cc} d_1 & d_2 \end{array} \\
\begin{array}{c} k_1 \\ k_2 \\ k_3 \end{array} & \left[\begin{array}{cc} f_{1,1} & f_{1,2} \\ f_{2,1} & f_{2,2} \\ f_{3,1} & f_{3,2} \end{array} \right]
\end{array}
$$

  – where each $f_{i,j}$ element stands for the frequency of term k_i in document d_j

Basic Concepts (cont.)

- Not all terms are equally useful for representing the document contents
  - *less frequent terms* allow identifying a narrower set of documents
- The *importance* of the index terms is represented by *weights* associated to them
  - Let
    - k_i be an index term
    - d_j be a document
    - w_{ij} be a weight associated with (k_i, d_j)
    - $\vec{d_j}=(w_{1,j}, w_{2,j}, \ldots, w_{t,j})$: an index term vector for the document d_j
    - $g_i(\vec{d_j})= w_{i,j}$
  - The weight w_{ij} quantifies the importance of the index term for describing the document semantic contents

Classic IR Models - Basic Concepts (cont.)

- Correlation of index terms
  - E.g.: computer and network
  - Consideration of such correlation information does not consistently improve the final ranking result
    - Complex and slow operations

- Important Assumption/Simplification
  - Index term weights are mutually independent ! (bag-of-words modeling)
  - However, the appearance of one word often attracts the appearance of the other (e.g., "Computer" and "Network")

The Boolean Model

- Simple model based on set theory and Boolean algebra

- A query is specified as boolean expressions with and, or, not operations (connectives)
  - Precise semantics, neat formalism and simplicity
  - Terms are either present or absent, i.e., $w_{ij} \in \{0,1\}$

- A query can be expressed as a disjunctive normal form (DNF) composed of conjunctive components
  - $\vec{q}_{dnf}$: the DNF for a query q
  - $\vec{q}_{cc}$: conjunctive components (binary weighted vectors) of $\vec{q}_{dnf}$

The Boolean Model (cont.)

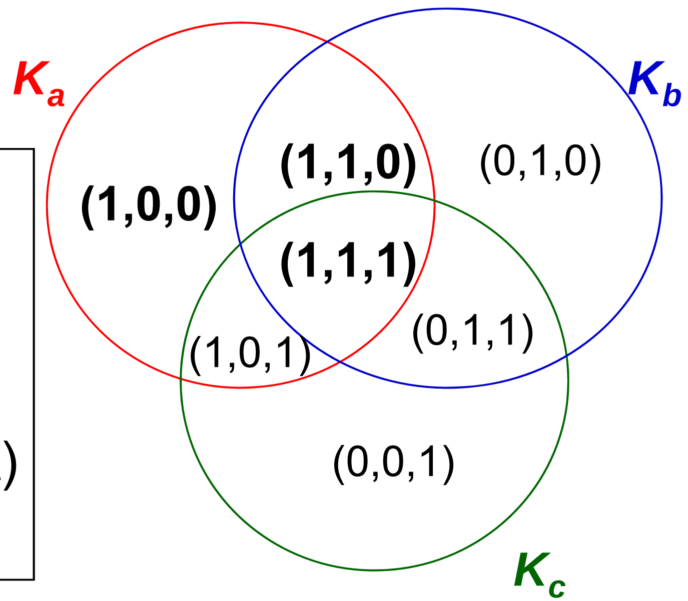
- For intance, a query [$q = k_a \wedge (k_b \vee \neg k_c)$] can be written as a DNF

$$\vec{q}_{dnf}=(1,1,1) \vee (1,1,0) \vee (1,0,0)$$

a canonical representation

conjunctive components
(binary weighted vectors)

Does $d= k_a \wedge k_b \wedge \neg k_c$ satisfy q ?



$k_a \wedge (k_b \vee \neg k_c)$
$=(k_a \wedge k_b) \vee (k_a \wedge \neg k_c)$
$= (k_a \wedge k_b \wedge k_c) \vee (k_a \wedge k_b \wedge \neg k_c)$
$\vee(k_a \wedge k_b \wedge \neg k_c) \vee(k_a \wedge \neg k_b \wedge \neg k_c)$
$= (k_a \wedge k_b \wedge k_c) \vee (k_a \wedge k_b \wedge \neg k_c) \vee(k_a \wedge \neg k_b \wedge \neg k_c)$
$\Rightarrow \vec{q}_{dnf}=(1,1,1) \vee (1,1,0) \vee (1,0,0)$

The Boolean Model (cont.)

- The similarity of a document d_j to the query q (i.e., premise of relevance)

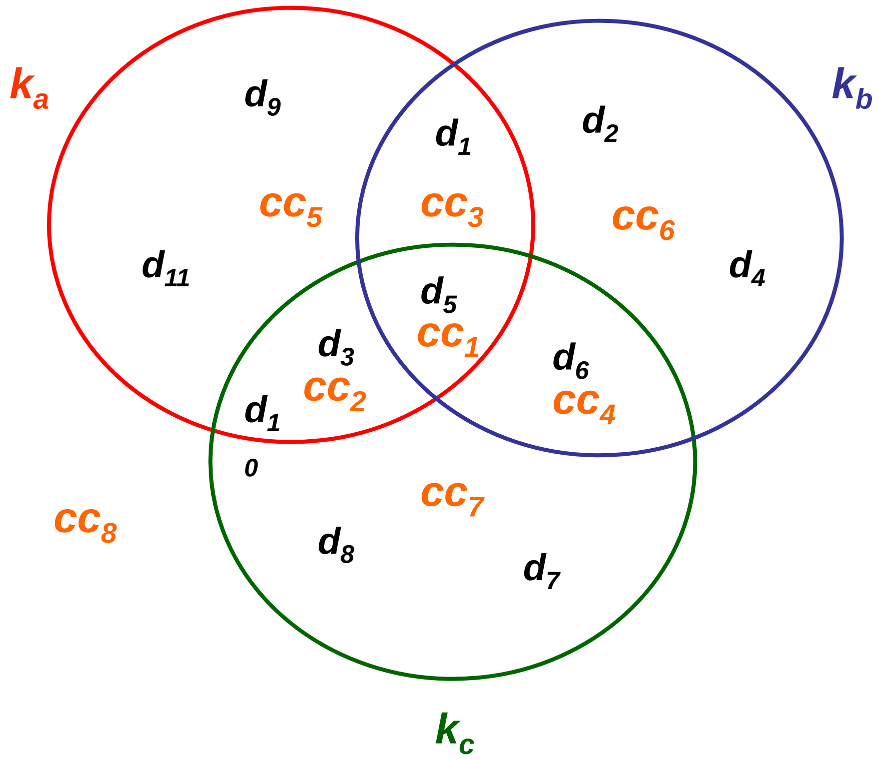
$$sim(d_j,q)=\begin{cases} 1: \text{if } \exists \vec{q}_{cc} \mid (\vec{q}_{cc} \in \vec{q}_{dnf} \wedge (\forall k_i,\ g_i(\vec{d}_j)=g_i(\vec{q}_{cc})) \\ 0: \text{otherwise} \end{cases}$$

A document is represented as
a conjunctive normal form

  - $sim(d_j,q)$=1 means that the document d_j is relevant to the query q
  - Each document d_j can be represented as a conjunctive component (vector)

Advantages of the Boolean Model

- Simple queries are easy to understand and relatively easy to implement (simplicity and neat model formulation)
- The dominant language (model) in commercial (bibliographic) systems until the WWW



$$
\begin{aligned}
cc_1 &= k_a\, k_b\, k_c\\
cc_2 &= k_a\, \overline{k_b}\, k_c\\
cc_3 &= k_a\, k_b\, \overline{k_c}\\
cc_4 &= \overline{k_a}\, k_b\, k_c\\
cc_5 &= k_a\, \overline{k_b}\, \overline{k_c}\\
cc_6 &= \overline{k_a}\, k_b\, \overline{k_c}\\
cc_7 &= \overline{k_a}\, \overline{k_b}\, k_c\\
cc_8 &= \overline{k_a}\, \overline{k_b}\, \overline{k_c}
\end{aligned}
$$

Drawbacks of the Boolean Model

- Retrieval based on binary decision criteria with no notion of partial matching (no term weighting)
  - No noton of a partial match to the query condition
  - No ranking (ordering) of the documents is provided (absence of a grading scale)
  - Term freqency counts in documents are not considered
  - Much more like a data retrieval model

Drawbacks of the Boolean Model (cont.)

- Information need has to be translated into a Boolean expression which most users find awkward
  - The Boolean queries formulated by the users are most often too simplistic (difficult to specify what is wanted)
- As a consequence, the Boolean model frequently returns either too few or too many documents in response to a user query
- However, the Boolean model is still dominant model with commercial document database systems

Term Weighting

- The terms of a document are not equally useful for describing the document contents
- In fact, there are index terms which are simply vaguer than others
- There are (occurrence) properties of an index term which are useful for evaluating the importance of the term in a document
- For instance, a word which appears in all documents of a collection is completely useless for retrieval tasks
  - However, deciding on the importance of a term for summarizing the contents of a document is not a trivial issue

Term Weighting (cont.)

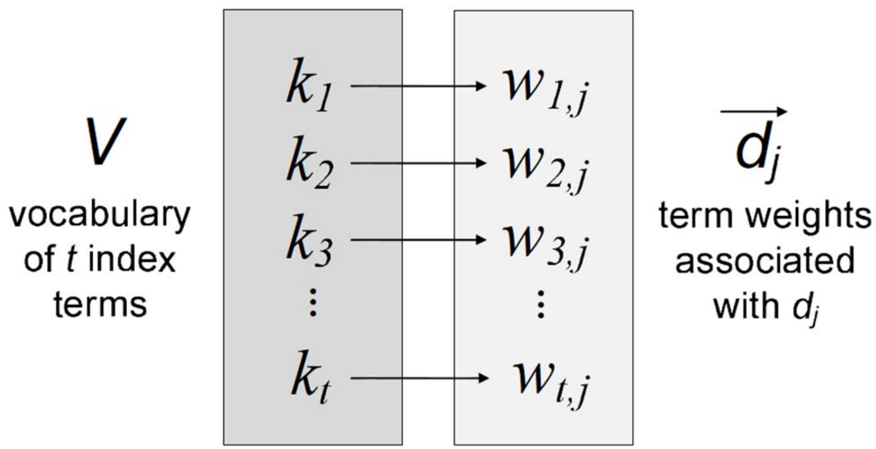
- To characterize term importance, we associate a weight $w_{i,j} > 0$ with each term k_i that occurs in the document d_j
  - If k_i that does not appear in the document d_j, then $w_{i,j} = 0$

- The weight $w_{i,j}$ quantifies the importance of the index term k_i for describing the contents of document d_j

- These weights are useful to compute a rank for each document in the collection with regard to a given query

Term Weighting (cont.)

- Let,
  - k_i be an index term and dj be a document
  - $V = \{k_1, k_2, ..., k_t\}$ be the set of all index terms
  - $w_{i,j} > 0$ be the weight associated with (k_i, d_j)
- Then we define $\vec{d_j} = (w_{1,j}, w_{2,j}, ..., w_{t,j})$ as **a weighted vector** that contains the weight $w_{i,j}$ of each term $k_i \in V$ in the document d_j



Term Weighting (cont.)

- The weights $w_{i,j}$ can be computed using the **frequencies of occurrence** of the terms within documents

- Let $f_{i,j}$ be the frequency of occurrence of index term k_i in the document d_j

- The **total frequency of occurrence** F_i of term k_i in the collection is defined as

$$F_i = \sum_{j=1}^{N} f_{i,j}$$

  – where N is the number of documents in the collection

Term Weighting (cont.)

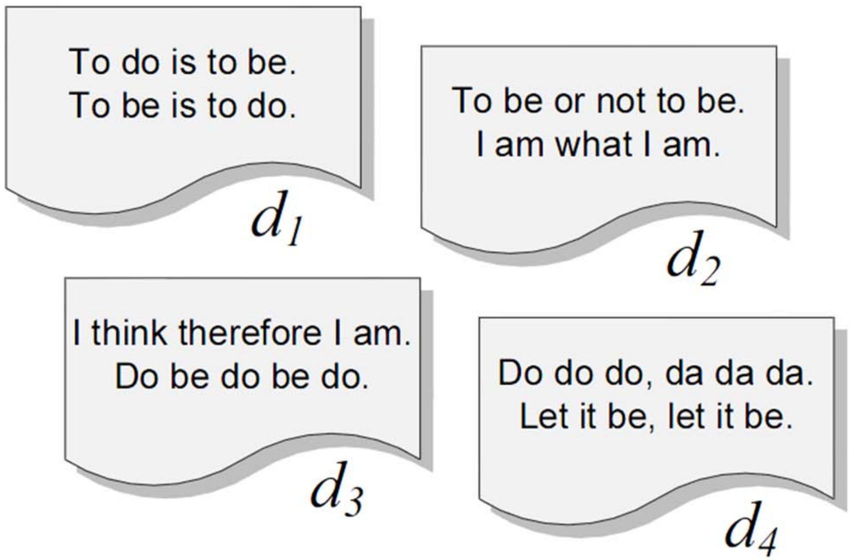
- The **document frequency** n_i of a term k_i is the number of documents in which it occurs
  - Notice that $n_i \leq F_i$

- For instance, in the document collection below, the values $f_{i,j}$, F_i and n_i associated with the term "*do*" are

$$f(do, d_1) = 2$$
$$f(do, d_2) = 0$$
$$f(do, d_3) = 3$$
$$f(do, d_4) = 3$$

$$F(do) = 8$$

$$n(do) = 3$$



Term-Term Correlation Matrix

- For classic information retrieval models, the index term weights are assumed to be **mutually independent**
  - This means that $w_{i,j}$ tells us nothing about $w_{i+1,j}$
- This is clearly a simplification because occurrences of index terms in a document are not uncorrelated
- For instance, the terms **computer** and **network** tend to appear together in a document about **computer networks**
  - In this document, the appearance of one of these terms attracts the appearance of the other
  - Thus, they are correlated and their weights should reflect this correlation

Term-Term Correlation Matrix (cont.)

- To take into account term-term correlations, we can compute a correlation matrix
- Let $\vec{M} = [m_{ij}]$ be a term-document matrix $t \times N$ where $m_{ij} = w_{i,j}$
- The matrix $\vec{C} = \vec{M} \cdot \vec{M}^t$ is a term-term correlation matrix
- Each element $c_{u,v} \in C$ expresses a correlation between terms k_u and k_v, given by

$$c_{uv} = \sum_{d_j} w_{u,j} \times w_{v,j}$$

  – Higher the number of documents in which the terms k_u and k_v co-occur, stronger is this correlation

Term-Term Correlation Matrix (cont.)

- Term-term correlation matrix for a sample collection

$$
\underbrace{
\begin{array}{cc}
 & \begin{array}{cc} d_1 & d_2 \end{array} \\
\begin{array}{c} k_1 \\ k_2 \\ k_3 \end{array} &
\left[\begin{array}{cc}
w_{1,1} & w_{1,2} \\
w_{2,1} & w_{2,2} \\
w_{3,1} & w_{3,2}
\end{array}\right] \\
 & \mathbf{M}
\end{array}
\times
\begin{array}{cc}
 & \begin{array}{ccc} k_1 & k_2 & k_3 \end{array} \\
\begin{array}{c} d_1 \\ d_2 \end{array} &
\left[\begin{array}{ccc}
w_{1,1} & w_{2,1} & w_{3,1} \\
w_{1,2} & w_{2,2} & w_{3,2}
\end{array}\right] \\
 & \mathbf{M}^T
\end{array}
}
$$

$$\Downarrow$$

$$
\begin{array}{cc}
 & \begin{array}{ccc} k_1 & k_2 & k_3 \end{array} \\
\begin{array}{c} k_1 \\ k_2 \\ k_3 \end{array} &
\left[\begin{array}{ccc}
w_{1,1}w_{1,1} + w_{1,2}w_{1,2} & w_{1,1}w_{2,1} + w_{1,2}w_{2,2} & w_{1,1}w_{3,1} + w_{1,2}w_{3,2} \\
w_{2,1}w_{1,1} + w_{2,2}w_{1,2} & w_{2,1}w_{2,1} + w_{2,2}w_{2,2} & w_{2,1}w_{3,1} + w_{2,2}w_{3,2} \\
w_{3,1}w_{1,1} + w_{3,2}w_{1,2} & w_{3,1}w_{2,1} + w_{3,2}w_{2,2} & w_{3,1}w_{3,1} + w_{3,2}w_{3,2}
\end{array}\right]
\end{array}
$$

  - Further, we can take advantage of factors such as term-term distances inside documents to improve the estimates of term-term correlations (see Chapter 5)

TF-IDF Weights

- Term frequency (TF)
- Inverse document frequency (IDF)

They are foundations (building blocks) of the most popular term weighting scheme in IR, called **TF-IDF**

Term Frequency (TF) Weights

- The simplest formulation is

$$tf_{i,j} = f_{i,j}$$

The frequency of occurrence of index term k_i im the document d_j

- A variant of *tf* weight used in the literature is

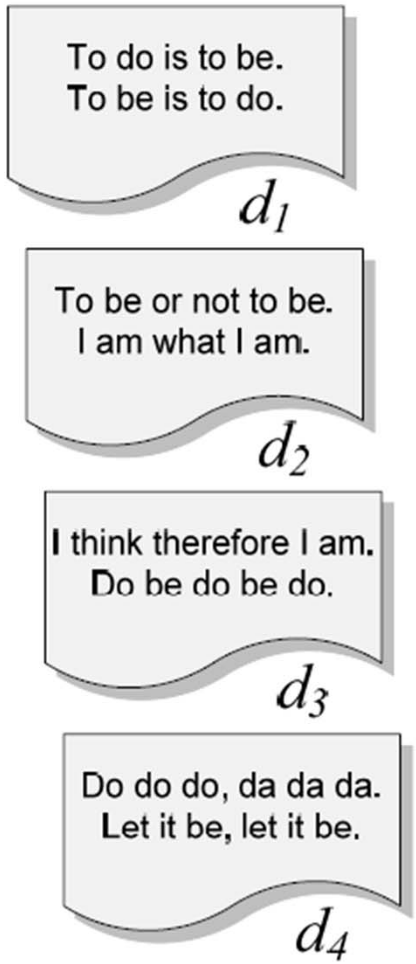
$$tf_{i,j} = \begin{cases} 1 + \log f_{i,j} & \text{if } \ f_{i,j} > 0 \\ 0 & \text{otherwise} \end{cases}$$

  – Where the log is taken in base 2

- The log expression is a the preferred form because it makes them directly comparable to *idf* weights, as we later discuss

Term Frequency (TF) Weights: An Example

- Log *tf* weights $tf_{i,j}$ for the example collection



Vocabulary		$tf_{i,1}$	$tf_{i,2}$	$tf_{i,3}$	$tf_{i,4}$
1	to	3	2	-	-
2	do	2	-	2.585	2.585
3	is	2	-	-	-
4	be	2	2	2	2
5	or	-	1	-	-
6	not	-	1	-	-
7	I	-	2	2	-
8	am	-	2	1	-
9	what	-	1	-	-
10	think	-	-	1	-
11	therefore	-	-	1	-
12	da	-	-	-	2.585
13	let	-	-	-	2
14	it	-	-	-	2

Inverse Document Frequency

- We call **document exhaustivity** the number of index terms assigned to a document
- The more index terms are assigned to a document, the higher is the probability of retrieval for that document
  - If too many terms are assigned to a document, it will be retrieved by queries for which it is not relevant
- **Optimal exhaustivity**: we can circumvent this problem by optimizing the number of terms per document
- Another approach is by weighting the terms differently, by exploring the notion of term specificity

Inverse Document Frequency (cont.)

- **Specificity** is a property of the term semantics
  - Term is more or less specific depending on its meaning
  - To exemplify, the term **beverage** is less specific than the terms **tea** and **beer**
  - We could expect that the term **beverage** occurs in more documents than the terms **tea** and **beer**
- Term specificity should be interpreted as a statistical rather than semantic property of the term
- **Statistical term specificity**: the inverse of the number of documents in which the term occurs

Inverse Document Frequency : Derivation

- Terms are distributed in a text according to **Zipf's Law**
- Thus, if we sort the vocabulary terms in decreasing order of document frequencies we have

$$n(r) \propto r^{-\alpha}$$

There is an inverse relationship between $n(r)$ and r

  - Where $n(r)$ refers to the r-th largest **document frequency** and α is an empirical constant
- That is, the **document frequency** of term k_i is an exponential function of its rank

$$n(r) = Cr^{-\alpha}$$

  - where C is a second empirical constant

Inverse Document Frequency : Derivation

- Setting α = 1 (simple approximation for English collections) and taking logs we have

$$\log n(r) = \log C - \log r$$

- For r = 1, we have $C = n(1)$, i.e., the value of C is the largest document frequency
  - This value (i.e., C's value) works as a normalization constant
- An alternative is to do the normalization assuming $C = N$, where N is the number of documents in the collection

$$\log r \approx \log N - \log n(r) = \log \frac{N}{n(r)}$$

Inverse Document Frequency : Derivation

- Let k_i be the term with the r-th largest document frequency, i.e., $n(r) = n_i$. Then,

$$IDF_i = \log \frac{N}{n_i}$$

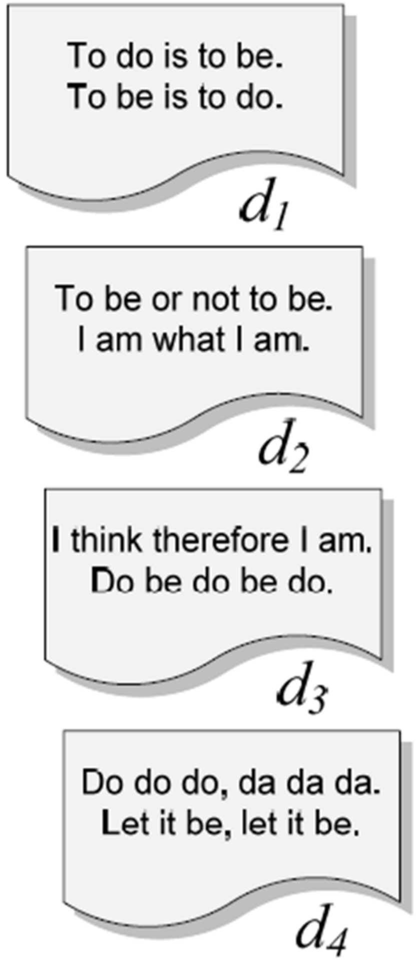
Sparck Jones, 1972

  - where idf_i is called the **inverse document frequency** of term k_i

- IDF provides a foundation for modern term weighting schemes and is used for ranking in almost all IR systems

Inverse Document Frequency : An Example

- IDF values for example collection



	term	n_i	$idf_i = \log(N/n_i)$
1	to	2	1
2	do	3	0.415
3	is	1	2
4	be	4	0
5	or	1	2
6	not	1	2
7	I	2	1
8	am	2	1
9	what	1	2
10	think	1	2
11	therefore	1	2
12	da	1	2
13	let	1	2
14	it	1	2

More on Inverse Document Frequency

- In a large real collection, we expect the most selective (discriminative) terms to be nouns or noun groups (a noun composed of various words)
- The least selective terms are usually article, conjunctions, and prepositions which are frequently referred to as **stop words**
- IDF weights provide a foundation for modern term weighting schemes and are used by almost any modern IR system

TF-IDF weighting scheme

Salton and Yang, 1973

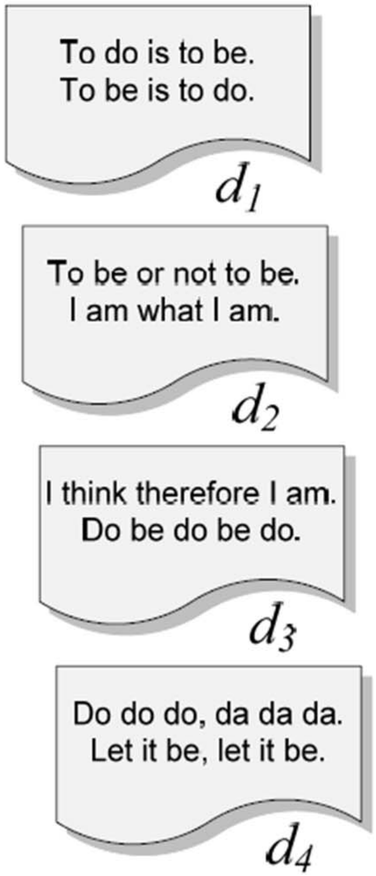
- The best known term weighting schemes use weights that combine **IDF factors** with **term frequencies**
- Let $w_{i,j}$ be the term weight associated with the term k_i and the document d_j
- Then, we define

$$w_{i,j} = \begin{cases} \left(1 + \log f_{i,j}\right) \times \log \dfrac{N}{n_i} & \text{if} \quad f_{i,j} > 0 \\ 0 & \text{otherwise} \end{cases}$$

  – Which is referred to as a **TF-IDF weighting scheme**

TF-IDF weighting scheme: An Example

- TF-IDF weights of all terms present in our example document collection



		d_1	d_2	d_3	d_4
1	to	3	2	-	-
2	do	0.830	-	1.073	1.073
3	is	4	-	-	-
4	be	-	-	-	-
5	or	-	2	-	-
6	not	-	2	-	-
7	I	-	2	2	-
8	am	-	2	1	-
9	what	-	2	-	-
10	think	-	-	2	-
11	therefore	-	-	2	-
12	da	-	-	-	5.170
13	let	-	-	-	4
14	it	-	-	-	4

Variants of TF-IDF

- Several variations of the above expression for TF-IDF weights are described in the literature
- For **TF weights**, five distinct variants are illustrated below

		tf weight
	binary	{0,1}
(frequency count)	raw frequency	$f_{i,j}$
	log normalization	$1 + \log f_{i,j}$
	double normalization 0.5	$0.5 + 0.5 \frac{f_{i,j}}{max_i f_{i,j}}$
	double normalization K	$K + (1 - K) \frac{f_{i,j}}{max_i f_{i,j}}$

Variants of TF-IDF (Cont.)

- Five distinct variants of **IDF weights**

	idf weight
unary	1
inverse frequency	$\log \frac{N}{n_i}$
inv frequency smooth	$\log(1 + \frac{N}{n_i})$
inv frequeny max	$\log(1 + \frac{max_i n_i}{n_i})$
probabilistic inv frequency	$\log \frac{N - n_i}{n_i}$

  - The log normalization variant uses the logarithm to assign diminishing increases to the weight (as the frequency increases), analogously to IDF weights
  - The probabilistic inverse frequency variant (would have negative values) arises from the classic probabilistic model, as discussed later on

Variants of TF-IDF (Cont.)

- Distinct combinations of TF variants and IDF variants yield various forms of TF-IDF weights
  - Recommended TF-IDF weighting schemes proposed by Salton and his colleague

weighting scheme	document term weight	query term weight
1	$f_{i,j} * \log \frac{N}{n_i}$	$(0.5 + 0.5 \frac{f_{i,q}}{max_i \ f_{i,q}}) * \log \frac{N}{n_i}$
2	$1 + \log f_{i,j}$	$\log(1 + \frac{N}{n_i})$
3	$(1 + \log f_{i,j}) * \log \frac{N}{n_i}$	$(1 + \log f_{i,q}) * \log \frac{N}{n_i}$

Salton & Buckley

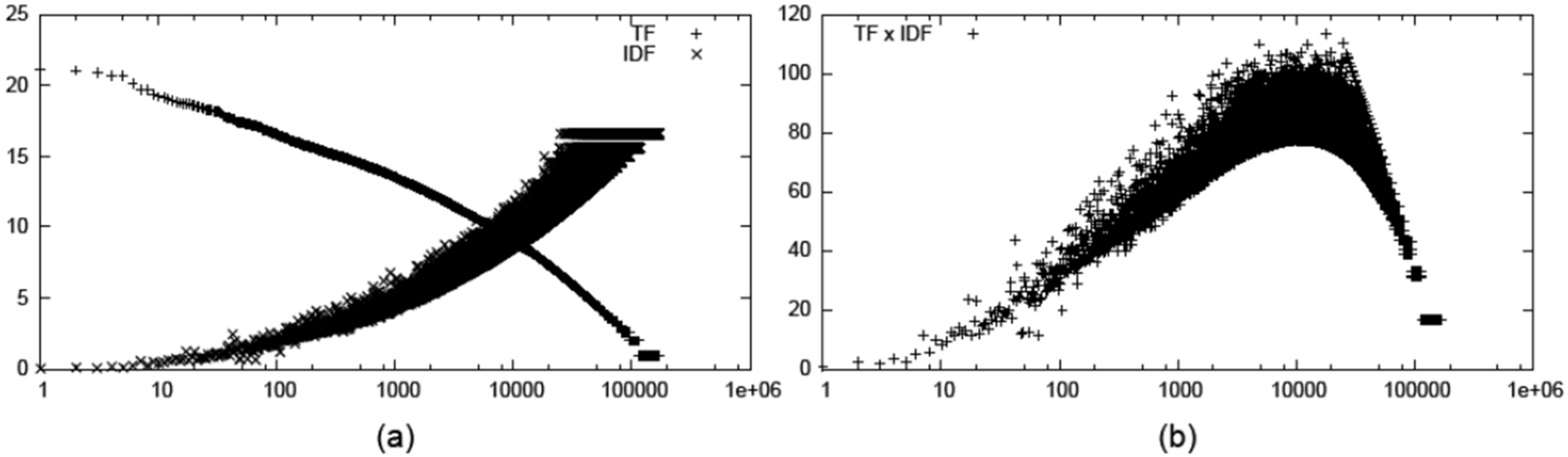
TF-IDF Properties

- Consider the TF, IDF, and TF-IDF weights for the Wall Street Journal reference collection
- To study their behavior, we would like to plot them together
- While IDF is computed over all the collection, TF is computed on a per document basis
  - Thus, we need a representation of TF based on all the collection, which is provided by the **term collection frequency** *TF_i*
- This reasoning leads to the following TF and IDF term weight

$$TF_i = 1 + \sum_{j=1}^{N} f_{i,j} \qquad IDF_i = \frac{N}{n_i}$$

TF-IDF Properties (Cont.)

- Plotting TF and IDF in logarithmic scale yields



  - Statistics are gathered from the *Wall Street Journal* collection
  - The horizontal axis corresponding the rank of each term according to TF
- We observe that TF and IDF weights present **power-law** behaviors that balance each other
- The terms of **intermediate IDF values** display maximum TF-IDF weights and are most interesting for ranking

In statistics, a power law is a functional relationship between two quantities, where one quantity varies as a power of another.

TF-IDF Properties (Cont.)

- Common terms (such as stopwords) and rare terms (such as foreign words or misspellings) are not of great value for ranking

Document Length Normalization

- Document sizes might vary widely
- This is a problem because longer documents are more likely to be retrieved by a given query
- To compensate for this undesired effect, we can divide the rank of each document by its length
- This procedure consistently leads to better ranking, and it is called **document length normalization**

Document Length Normalization (cont.)

- Methods of document length normalization depend on the representation adopted for the documents:
  - **Size in bytes**: consider that each document is represented simply as a stream of bytes
  - **Number of words**: each document is represented as a single string, and the document length is the number of words in it (compute document lengths at a syntactic level, which includes more semantics)
  - **Vector norms**: documents are represented as vectors of weighted terms

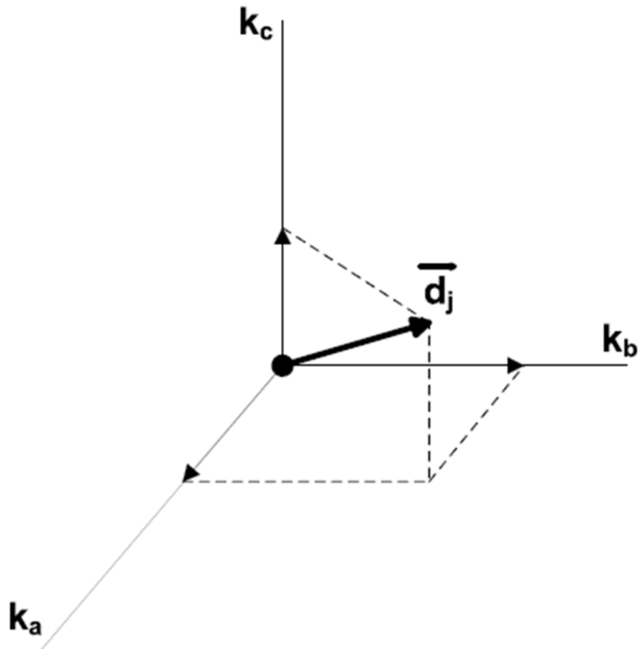
$$\vec{d_j} = (w_{1,j}, w_{2,j}, ..., w_{t,j})$$

    - The weights of the various terms are taken into consideration)

$$|\vec{d_j}| = \sqrt{\sum_i^t w_{i,j}^2}$$

Document Length Normalization (cont.)

- Documents represented as vectors of weighted terms
  - Each term of a collection is associated with **an orthonormal unit vector** $\vec{k_i}$ in a t-dimensional space
  - For each term k_i of a document d_j is associated the term vector
  - component $w_{i,j} \times \vec{k_i}$



Document Length Normalization (cont.)

- The document representation $\vec{d_j}$ is a vector composed of all its term vector components

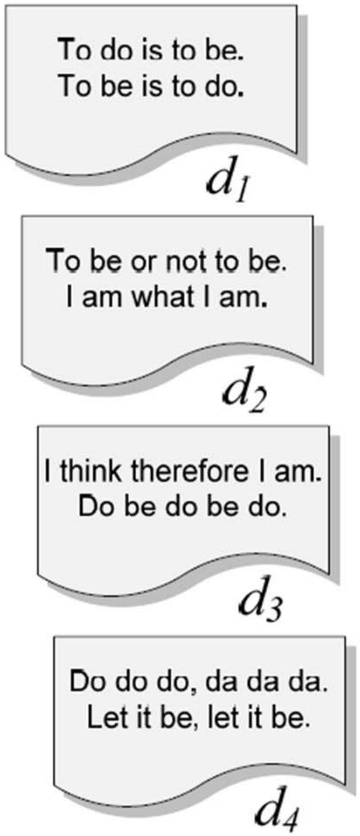
$$\vec{d}_j = \left(w_{1,j}, w_{2,j}, \ldots, w_{t,j}\right)$$

- The document length is given by the norm of this vector, which is computed as follows

$$\left|\vec{d}_j\right| = \sqrt{\sum_{i=1}^{t} w_{i,j}^2}$$

Document Length Normalization (cont.)

- Three variants of document lengths for the example
- collection



	d_1	d_2	d_3	d_4
size in bytes	34	37	41	43
number of words	10	11	10	12
vector norm	5.068	4.899	3.762	7.738

The Vector Model

SMART system
Cornell U., 1968

- Also called *Vector Space Model (VSM)*

- Some perspectives
  - Use of *binary weights* is too limiting
  - *Non-binary weights* provide consideration for partial matches
  - These term weights are used to compute a *degree of similarity* between a query and each document
  - Ranked set of documents provides better matching for user information need

The Vector Model (cont.)

- Definition:
  - w_{ij} > =0 whenever $k_i \in d_j$
  - w_{iq} >= 0 whenever $k_i \in q$
  - document vector $\vec{d_j} = (w_{1j}, w_{2j}, ..., w_{tj})$
  - query vector $\vec{q} = (w_{1q}, w_{2q}, ..., w_{tq})$
  - To each term k_i is associated a unitary (basis) vector $\vec{u_i}$
  - The unitary vectors $\vec{u_i}$ and $\vec{u_s}$ are assumed to be **orthonormal** (i.e., index terms are assumed to occur independently within the documents)

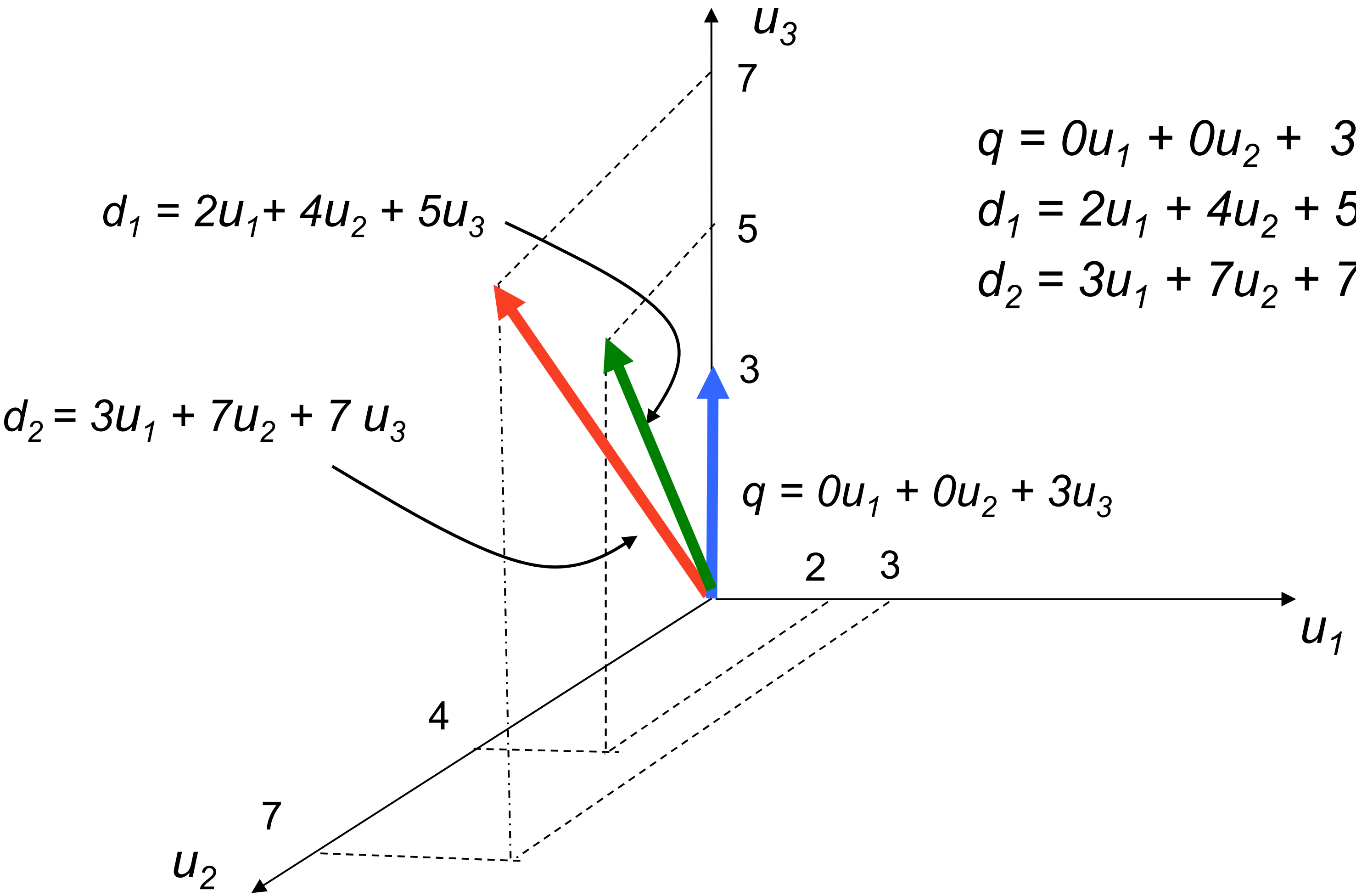
totally t terms in the vocabulary

- The t unitary vectors $\vec{u_i}$ form an orthonormal basis for a t-dimensional space
  - Queries and documents are represented as weighted vectors

The Vector Model (cont.)

- How to measure the degree of similarity
  - Distance, angle or projection?

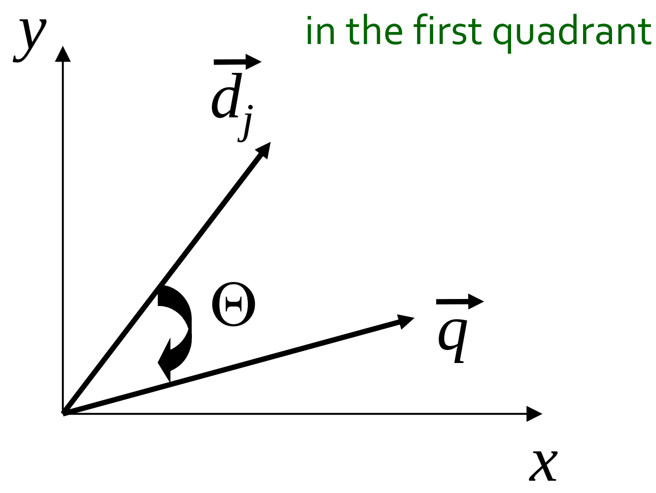
$q = 0u_1 + 0u_2 + 3u_3$
$d_1 = 2u_1 + 4u_2 + 5u_3$
$d_2 = 3u_1 + 7u_2 + 7u_3$



The Vector Model (cont.)

- The similarity of a document d_j to the query q

$$
\begin{aligned}
& sim(d_j, q) \\
&= \text{cosine}(\Theta) \\
&= \frac{\vec{d}_j \bullet \vec{q}}{|\vec{d}_j| \times |\vec{q}|} \\
&= \frac{\sum_{i=1}^{t} w_{i,j} \times w_{i,q}}{\sqrt{\sum_{i=1}^{t} w_{i,j}^2} \times \sqrt{\sum_{i=1}^{t} w_{i,q}^2}}
\end{aligned}
$$



Document length normalization

The same for documents, can be discarded ⇒ Won't affect the final ranking

(if discarded, equivalent to the projection of the query on the document vector)

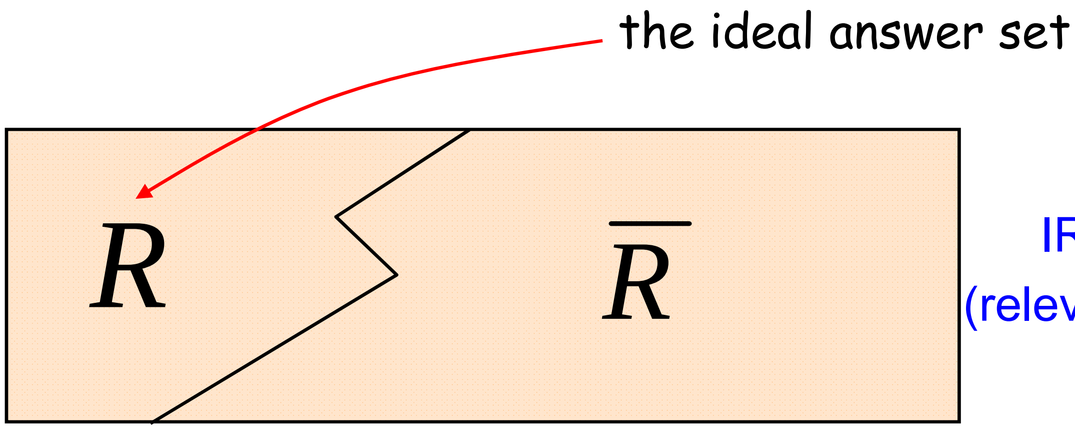
– Establish a threshold on $sim(d_j, q)$ and retrieve documents with a degree of similarity above the threshold

The Vector Model (cont.)

- Degree of similarity ⇨ Relevance
  - Usually, w_{ij} > =*0* & w_{iq} >= *0*
    - Cosine measure ranges between 0 and 1
  - $sim(d_j, q) \approx 1$ ⇨ highly relevant !
  - $sim(d_j, q) \approx 0$ ⇨ almost irrelevant !

The Vector Model (cont.)

- The role of index terms



Document collection

IR as a binary clustering (relevant/non-relevant) problem

- Which index terms (features) better describe the relevant class
  - Intra-cluster similarity (TF-factor)
  - Inter-cluster dissimilarity (IDF-factor)

balance between these two factors

The Vector Model (cont.)

- The vector model with **TF-IDF weights** is a good ranking strategy with *general* collections, for example

$$w_{i,q} = \left(1 + \log f_{i,q}\right) \times \log\left(\frac{N}{n_i}\right)$$

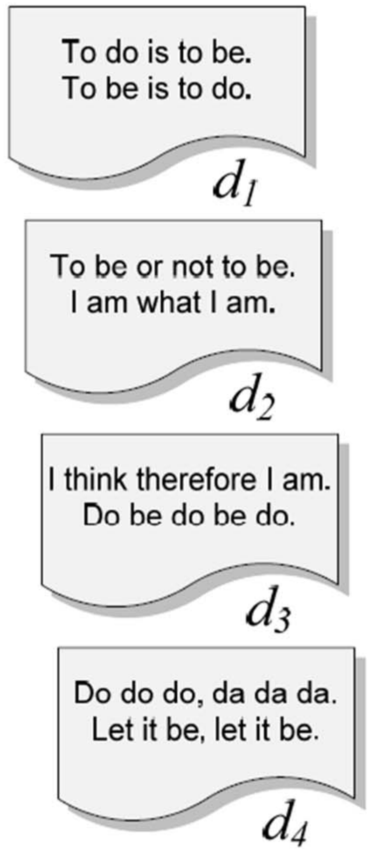
$$w_{i,j} = \left(1 + \log f_{i,j}\right) \times \log\left(\frac{N}{n_i}\right)$$

  - These equations should only be applied for values of term frequency greater than zero
  - If the term frequency is zero, the respective weight is also zero

- The vector model is usually as good as the known ranking alternatives. It is also simple and fast to compute

The Vector Model (cont.)

- Document ranks computed by the Vector model for the
- query “**to do**” (see TF-IDF weight values in Slide 49)



doc	rank computation	rank
d_1	$\frac{1*3+0.415*0.830}{5.068}$	0.660
d_2	$\frac{1*2+0.415*0}{4.899}$	0.408
d_3	$\frac{1*0+0.415*1.073}{3.762}$	0.118
d_4	$\frac{1*0+0.415*1.073}{7.738}$	0.058

The Vector Model (cont.)

- Experimental Results on TDT Chinese collections
  - Mandarin Chinese broadcast news
  - Measured in *mean* Average Precision (*m*AP)
  - ACM TALIP (2004)

Retrieval Results for the Vector Space Model

Average Precision		Word-level		Syllable-level	
		S(N), *N*=1	*S(N)*, *N*=1~2	*S(N)*, *N*=1	*S(N)*, *N*=1~2
TDT-2 (Dev.)	TD	0.5548	0.5623	0.3412	0.5254
	SD	0.5122	0.5225	0.3306	0.5077
TDT-3 (Eval.)	TD	0.6505	0.6531	0.3963	0.6502
	SD	0.6216	0.6233	0.3708	0.6353

$$R(q,d)=\sum_{j} w_j \cdot R_j(\vec{q}_j,\vec{d}_j),$$

types of index terms

The Vector Model (cont.)

- Advantages
  - *Term-weighting* improves quality of the answer set
  - *Partial matching* allows retrieval of docs that approximate the query conditions
  - Cosine ranking formula sorts documents according to degree of similarity to the query
  - Document normalization is naturally built-in into the ranking

- Disadvantages
  - Assumes mutual independence of index terms
    - Not clear that this is bad though (??): leveraging term dependencies is challenging and might lead to poor results, if not done appropriately

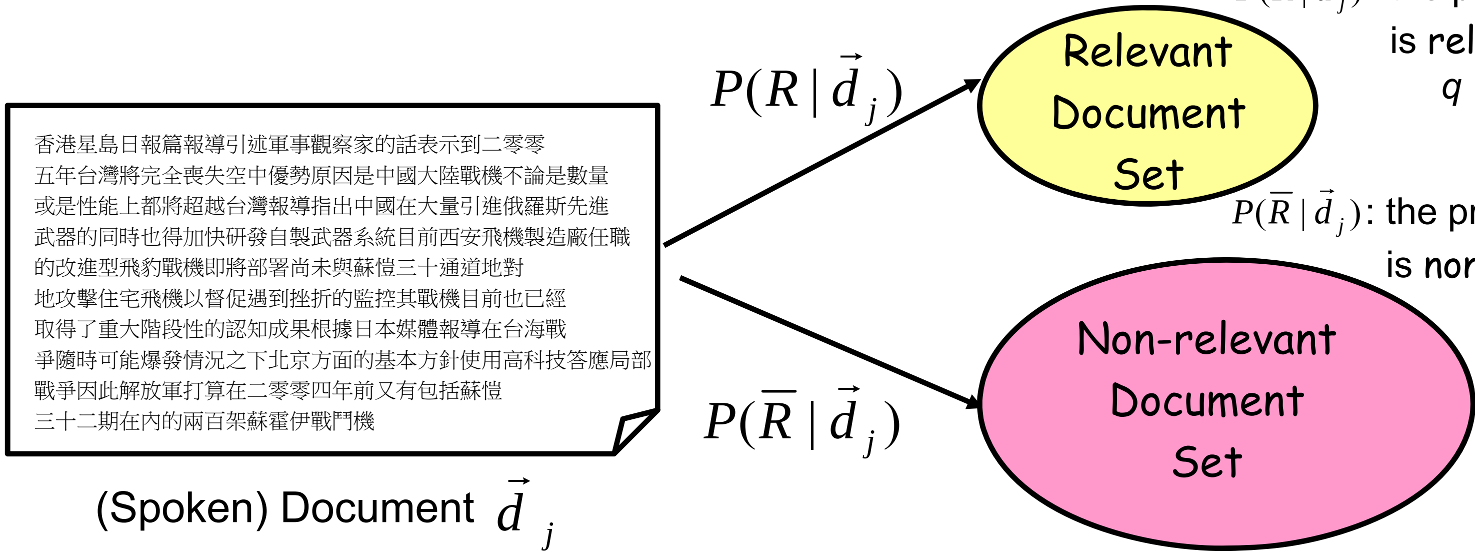
The Probabilistic Model

Roberston & Sparck Jones 1976

- Known as the Binary Independence Retrieval (BIR) model
  - “**Binary**”: all weights of index terms are binary (0 or 1)
  - “**Independence**”: index terms are independent !
- Capture the IR problem using a probabilistic framework
  - Bayes' decision rule

The Probabilistic Model (cont.)

- Retrieval is modeled as a *classification* process
  - Two classes for each query: the relevant or non-relevant documents



$P(R \mid \vec{d}_j)$: the prob. that the doc. d_j is relevant to the query q

$P(\overline{R} \mid \vec{d}_j)$: the prob. that the doc. d_j is non-relevant to the query q

The Probabilistic Model (cont.)

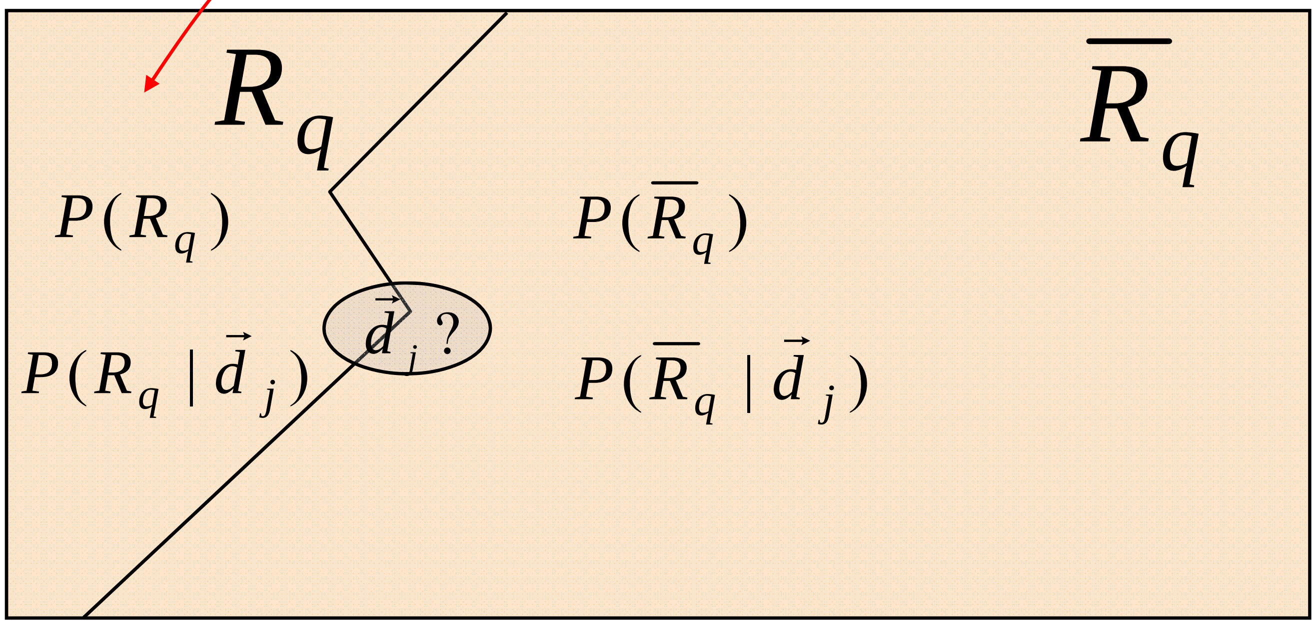
- Given a user query, there is an *ideal answer set*
  - Contain exactly the relevant documents and no others
  - The querying process as a specification of the properties of this ideal answer set (R_q)
- Problem: what are these properties?
  - Only the *semantics of* *index terms* can be used to characterize these properties
- ***Guess at the beginning*** what they could be
  - I.e., an initial guess for the preliminary probabilistis description of ideal answer set
- ***Improve/Refine the probabilistic description of the answer set by iterations/interations***
  - Without (or with) the assistance from a human subject

The Probabilistic Model (cont.)

- How to improve the probabilistic description of the ideal answer set ?

$$P(R_q \mid \vec{d}_j) > P(\overline{R}_q \mid \vec{d}_j)$$

the ideal answer set



Document Collection

The Probabilistic Model (cont.)

- Given a particular document d_j , calculate the probability of belonging to the relevant class, retrieve if greater than probability of belonging to non-relevant class

$$P(R_q \mid \vec{d}_j) > P(\overline{R}_q \mid \vec{d}_j)$$

Bayes' Decision Rule

- The similarity of a document d_j to the query q

$$sim(d_j, q) = \frac{P(R_q \mid \vec{d}_j)}{P(\overline{R}_q \mid \vec{d}_j)}$$

Likelihood/Odds Ratio Test

Bayes' Theory

$$= \frac{P(\vec{d}_j \mid R_q)P(R_q)}{P(\vec{d}_j \mid \overline{R}_q)P(\overline{R}_q)} \approx \frac{P(\vec{d}_j \mid R_q)}{P(\vec{d}_j \mid \overline{R}_q)}$$

The same for all documents

$\geq \tau$?

if so, retrieved !

The Probabilistic Model (cont.)

- Explanation
  - $P(R_q)$: the prob. that a doc randomly selected form the entire collection is relevant to the query *q*
  - $P(\vec{d}_j \mid R_q)$: the prob. that the doc d_j is **relevant** to the query *q* (selected from the relevant doc set *R*)

- Further assume **independence** of index terms

$$sim\left(d_j, q\right) \approx \frac{P(\vec{d}_j \mid R_q)}{P(\vec{d}_j \mid \overline{R}_q)}$$

$$\approx \frac{\left[\prod_{g_i(\vec{d}_j)=1} P(k_i \mid R_q)\right]\left[\prod_{g_i(\vec{d}_j)=0} P(\overline{k}_i \mid R_q)\right]}{\left[\prod_{g_i(\vec{d}_j)=1} P(k_i \mid \overline{R}_q)\right]\left[\prod_{g_i(\vec{d}_j)=0} P(\overline{k}_i \mid \overline{R}_q)\right]}$$

$P(k_i \mid R_q)$: prob. that k_i is present in a doc randomly selected form the set *R*

$P(\overline{k}_i \mid R_q)$: prob. that k_i is not present in a doc randomly selected form the set *R*

$$P(k_i \mid R_q) + P(\overline{k}_i \mid R_q) = 1$$

The Probabilistic Model (cont.)

- Further assume *independence* of index terms
  - Another representation

$$sim(d_j, q) \approx \frac{\prod_{i=1}^{t}\left[P(k_i \mid R_q)^{g_i(\vec{d}_j)} P(\bar{k}_i \mid R_q)^{1-g_i(\vec{d}_j)}\right]}{\prod_{i=1}^{t}\left[P(k_i \mid \bar{R}_q)^{g_i(\vec{d}_j)} P(\bar{k}_i \mid \bar{R}_q)^{1-g_i(\vec{d}_j)}\right]}$$

  - Take logarithms

$$sim(d_j, q) \approx \log \frac{\prod_{i=1}^{t}\left[P(k_i \mid R_q)^{g_i(\vec{d}_j)} P(\bar{k}_i \mid R_q)^{1-g_i(\vec{d}_j)}\right]}{\prod_{i=1}^{t}\left[P(k_i \mid \bar{R}_q)^{g_i(\vec{d}_j)} \left(P(\bar{k}_i \mid \bar{R}_q)\right)^{1-g_i(\vec{d}_j)}\right]}$$

The same for all documents!

$P(k_i \mid R_q) + P(\bar{k}_i \mid R_q) = 1$

$P(k_i \mid \bar{R}_q) + P(\bar{k}_i \mid \bar{R}_q) = 1$

$$= \sum_{i=1}^{t} g_i(\vec{d}_j) \log \frac{P(k_i \mid R_q) P(\bar{k}_i \mid \bar{R}_q)}{P(k_i \mid \bar{R}_q) P(\bar{k}_i \mid R_q)} + \cancel{\sum_{i=1}^{t} \log \frac{P(\bar{k}_i \mid R_q))}{P(\bar{k}_i \mid \bar{R}_q)}}$$

$$= \sum_{i=1}^{t} g_i(\vec{d}_j)\left[\log \frac{P(k_i \mid R_q)}{1 - P(k_i \mid R_q)} + \log \frac{1 - P(k_i \mid \bar{R}_q)}{P(k_i \mid \bar{R}_q)}\right]$$

The Probabilistic Model (cont.)

- Further assume **independence** of index terms
  - Use term weighting $w_{i,q} \times w_{i,j}$ to replace $g_i(\vec{d}_j)$

$$sim\left(d_j, q\right) \approx \sum_{i=1}^{t} g_i\left(\vec{d}_j\right)\left[\log \frac{P(k_i \mid R_q)}{1 - P(k_i \mid R_q)} + \log \frac{1 - P(k_i \mid \overline{R}_q)}{P(k_i \mid \overline{R}_q)}\right]$$

$$\approx \sum_{i=1}^{t} w_{i,q} \times w_{i,j} \times \left[\log \frac{P(k_i \mid R_q)}{1 - P(k_i \mid R_q)} + \log \frac{1 - P(k_i \mid \overline{R}_q)}{P(k_i \mid \overline{R}_q)}\right]$$

Binary weights (0 or 1) are used here

R_q is not known at the beginning

⇨ How to compute $P(k_i \mid R_q)$ and $P(k_i \mid \overline{R}_q)$

The Probabilistic Model (cont.)

- Initial Assumptions
  - $P(k_i \mid R_q) = 0.5$:is constant for all indexing terms
  - $P(k_i \mid \overline{R}_q) = \frac{n_i}{N}$:approx. by distribution of index terms among all doc in the collection, i.e. the document frequency of indexing term k_i (Suppose that $|\overline{R}|>>|R|$, $N \approx |\overline{R}|$)

  (n_i: no. of doc that contain k_i. N : the total doc no.)

- Re-estimate the probability distributions
  - Use the initially retrieved and ranked Top D documents

$$P(k_i \mid R_q) = \frac{D_i}{D}$$

$$P(k_i \mid \overline{R}_q) = \frac{n_i - D_i}{N - D}$$

D_i : the no. of documents in D that contain k_i

The Probabilistic Model (cont.)

- Handle the problem of "zero" probabilities
  - Add constants as the adjust constant

$$P(k_i \mid R_q) = \frac{D_i + 0.5}{D + 1}$$

$$P(k_i \mid \overline{R}_q) = \frac{n_i - D_i + 0.5}{N - D + 1}$$

  - Or use the information of document frequency

$$P(k_i \mid R_q) = \frac{D_i + \frac{n_i}{N}}{D + 1}$$

$$P(k_i \mid \overline{R}_q) = \frac{n_i - D_i + \frac{n_i}{N}}{N - D + 1}$$

The Probabilistic Model (cont.)

- Advantages
  - Documents are ranked in decreasing order of probability of relevance (optimality)

- Disadvantages
  - Need to guess initial estimates for $P(k_i \mid R)$
  - Estimate the characteristics of the relevant class/set R through user-identified examples of relevant docs (without true training data)
  - All weights are binary: the method does not take into account *tf* and *idf* factors
  - Independence assumption of index terms
  - The lack of document length normalization

More advanced variations of the probabilistic models, such as the BM-25 model, correct these deficiencies to yield improved retrieval.

Brief Comparisons of Classic Models

- Boolean model does not provide for *partial matches* and is considered to be the weakest classic model

- Salton and Buckley did a series of experiments that indicated that, in general, the vector model outperforms the probabilistic model with *general collections*
  - This also seems to be the dominant thought among researchers and practitioners of IR
  - The vector model, whose weighting scheme is firmly grounded on information theory, provides a simple yet effective ranking formula for general collections