

# Retrieval Performance Evaluation

## - Measures



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Reference:

1. *Modern Information Retrieval*, chapter 3

# Introduction

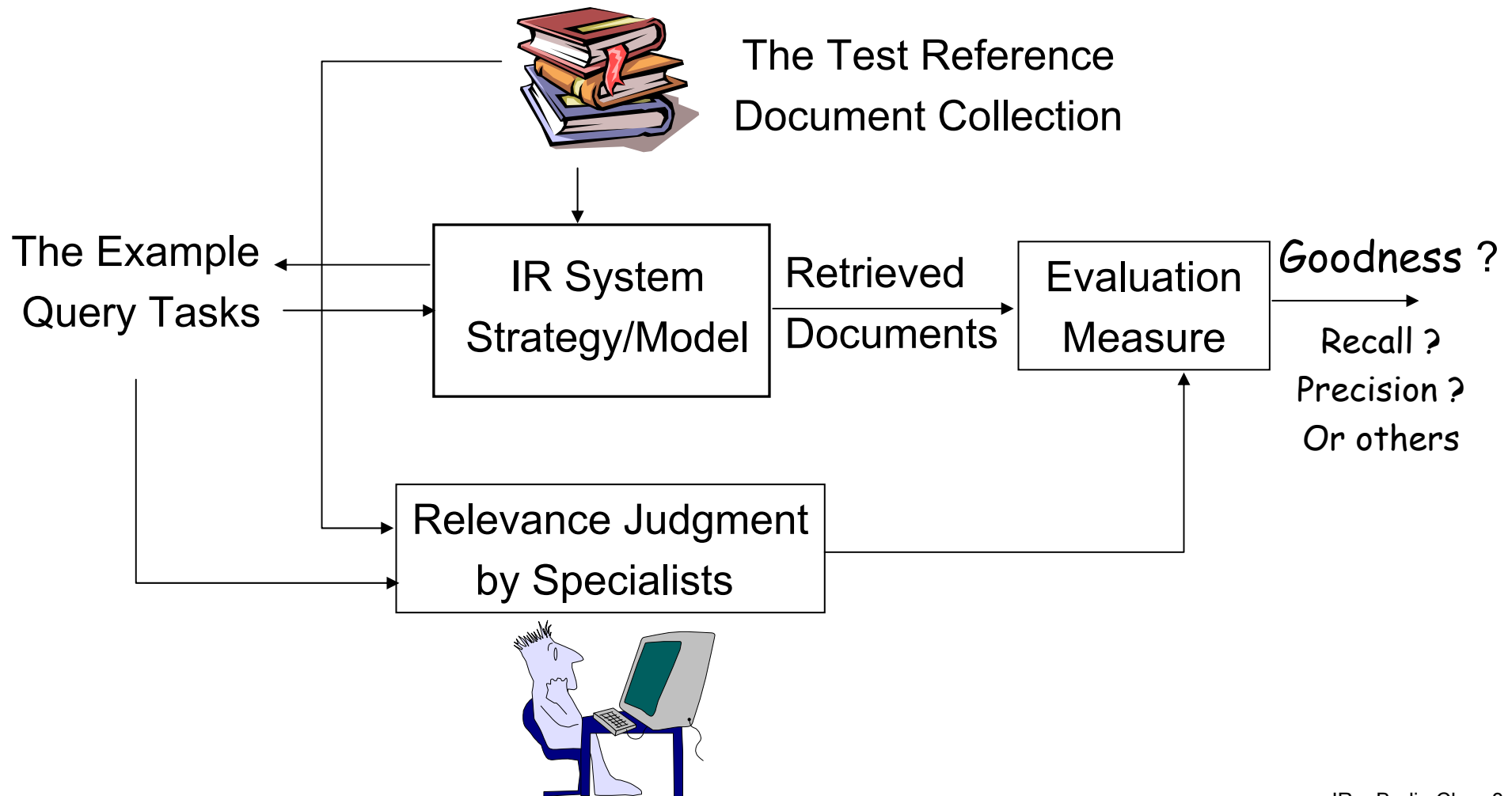
- Functional analysis
  - Functionality test or error analysis instead
- Performance evaluation
  - E.g.: **Data retrieval system**
    - The shorter the response time, the smaller the space used, the better the system is
    - Tradeoff between time and space
- **Retrieval** performance evaluation
  - E.g.: **information retrieval system**
    - Relevance of retrieved documents is important, besides time and space (quality of the answer set)
  - Discussed here !



Different objectives

# Introduction (cont.)

- **Retrieval** performance evaluation (cont.)



# Introduction (cont.)

- The Test Reference Collection
  - A collection of documents
  - A set of example information requests (queries)
  - A set of relevant documents for each information request
- Evaluation measure
  - Qualify the similarity between the set of documents retrieved and the set of relevant documents provided (by the specialists)
  - Provide an estimation of the **goodness** of the retrieval strategy

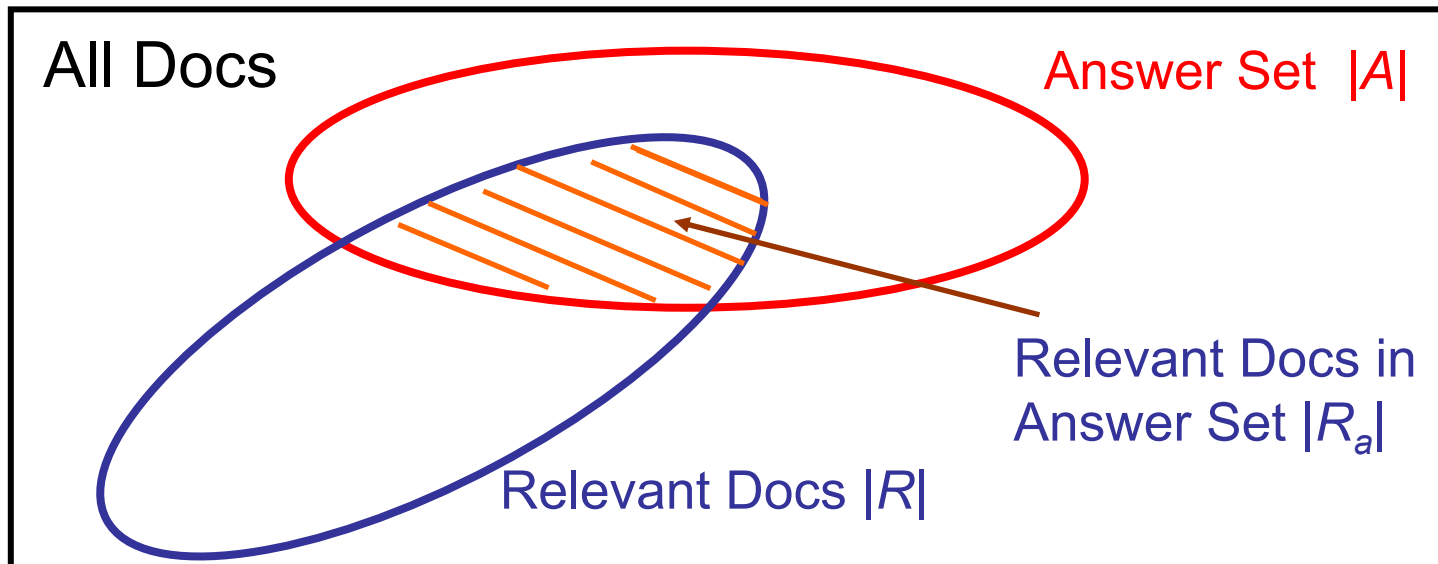
# Batch and Interactive Mode

## Consider retrieval performance evaluation

- Batch mode (laboratory experiments)
  - The user submits a query and receives an answer back
  - **Measure:** the quality of the generated answer set
  - Still the dominant evaluation (**Discussed here !**)
    - Main reasons: repeatability and scalability
- Interactive mode (real life situations)
  - The user specifies his information need through a series of interactive steps with the system
  - **Measure:** user effort, interface design, system's guidance, session duration
  - Get a lot more attention in 1990s

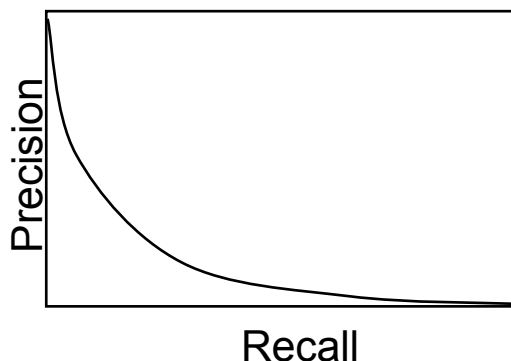
# Recall and Precision

- Recall (  $\frac{|R_a|}{|R|}$  )
  - The fraction of the relevant documents which has been retrieved
- Precision (  $\frac{|R_a|}{|A|}$  )
  - The fraction of the retrieved documents which is relevant



## Recall and Precision (cont.)

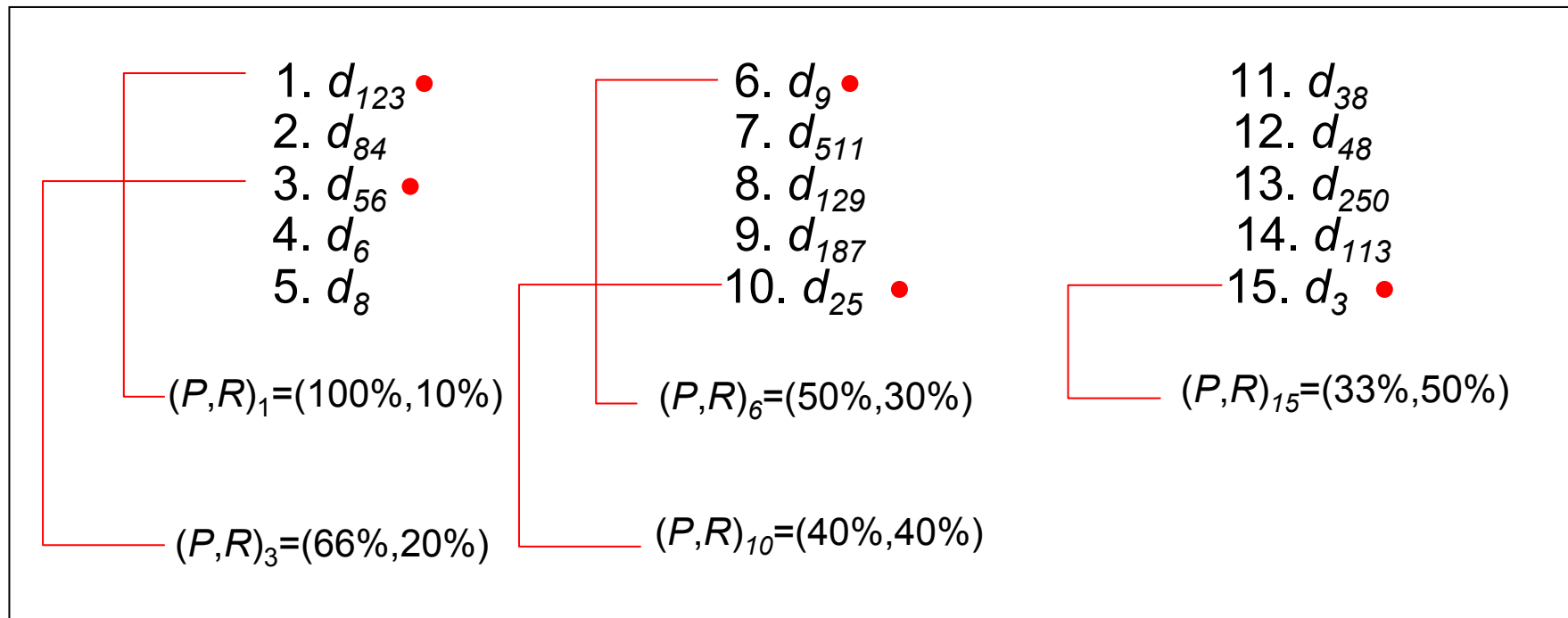
- Recall and precision **assume that all the documents in the answer set have been examined (or seen)**
- However, the user is not usually presented with all the documents in the answer set  $A$  at once
  - Sort the document in  $A$  according to a degree of relevance
  - Examine the ranked list starting from the top document (increasing in recall, but decreasing in precision)
    - Varying of recall and precision measures
    - A precision versus recall curve can be plotted



# Recall and Precision (cont.)

- Example 3.2

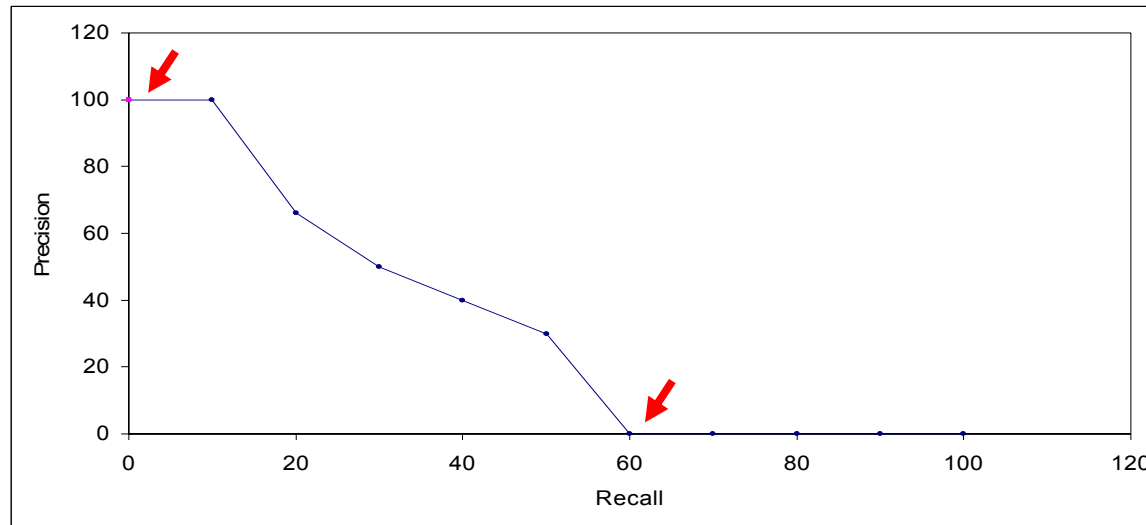
- $R_q = \{d_3, d_5, d_9, d_{25}, d_{39}, d_{44}, d_{56}, d_{71}, d_{89}, d_{123}\}$ 
  - Ten relevant documents, five included in Top 15
- A ranking of the documents for the given query  $q$





# Recall and Precision (cont.)

- Example 3.2 (count.)



- The precision versus recall curve is usually plotted based on 11 standard recall levels: 0%, 10%, ..., 100%
- In this example
  - The precisions for recall levels higher than 50% drop to 0 because no relevant documents were retrieved
  - There was an interpolation for the recall level 0%

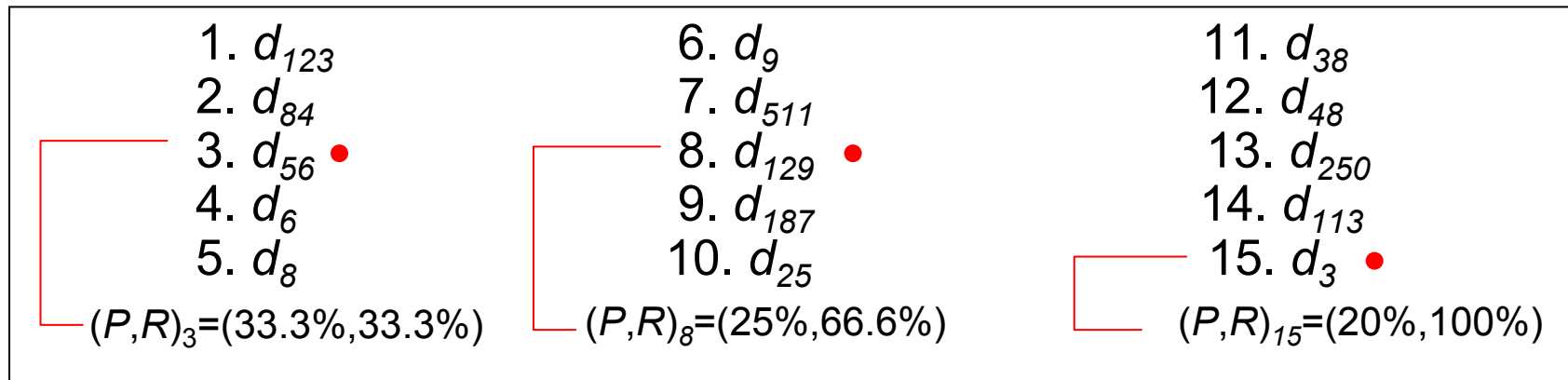
# Interpolated Recall-Precision Curve

- Since the recall levels for each query might be distinct from the 11 standard recall levels
  - Utilization of an interpolation procedure is necessary !

Salton, 1983

- Example 3.3

- $R_q = \{d_3, d_{56}, d_{129}\}$ 
  - Three relevant documents



- How about the precisions at recall levels 0%, 10%, ..., 90%

# Interpolated Recall-Precision Curve (cont.)

- Interpolated Precisions at standard recall levels

$$\bar{P}(r_j) = \max_{r_j \leq r \leq r_{j+1}} P(r)$$

– the  $j$ -th standard recall level (e.g.,  $r_5$  is recall level 50%)

- Example 3.3 (cont.)

$$(P, R)_3 = (33.3\%, 33.3\%)$$

$$(P, R)_8 = (25\%, 66.6\%)$$

$$(P, R)_{15} = (20\%, 100\%)$$

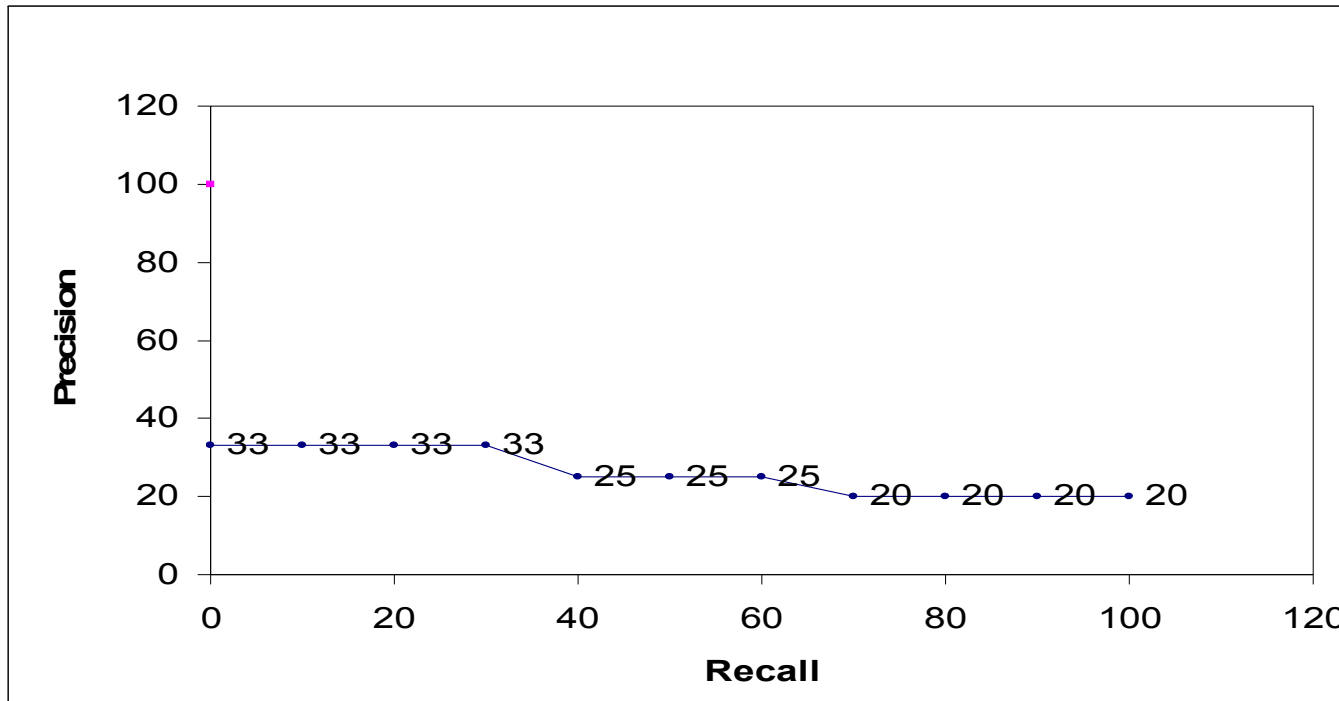
Precision	Recall
33.3%	0%
33.3%	10%
33.3%	20%
33.3%	30%
25%	40%
25%	50%
25%	60%
20%	70%
20%	80%
20%	90%
20%	100%

$$\bar{P}_i(r_j) = \max_{r_j \leq r \leq r_{j+1}} P_i(r)$$

query  $i$

# Interpolated Recall-Precision Curve (cont.)

- Example 3.3 (cont.)
  - Interpolated precisions at 11 standard recall levels



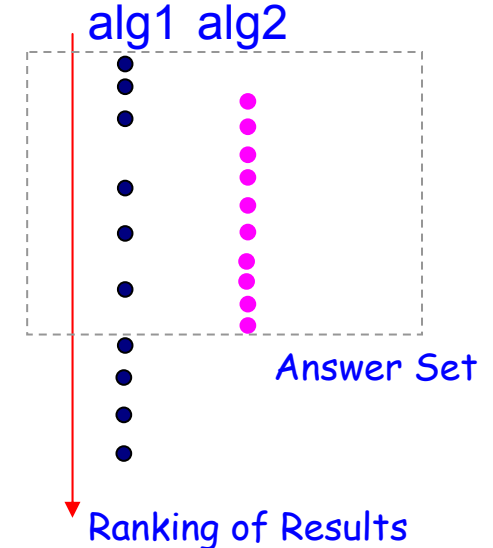
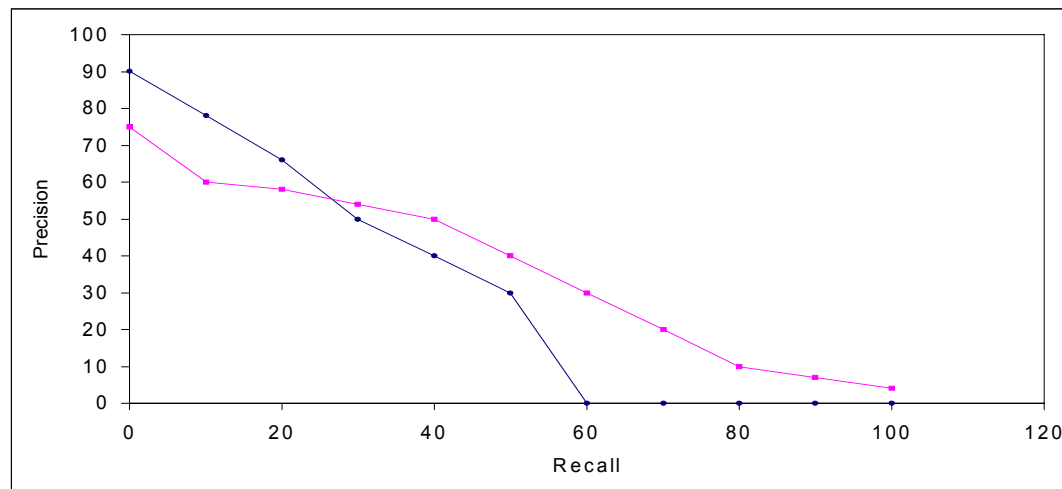
# Interpolated Recall-Precision Curve (cont.)

- Evaluate (average) the retrieval performance over all queries

$$\bar{P}_{all}(r_j) = \frac{1}{N_q} \sum_{i=1}^{N_q} \bar{P}_i(r_j)$$

On different recall levels

- Example 3.4: average interpolated recall-precision curves for two distinct retrieval algorithms



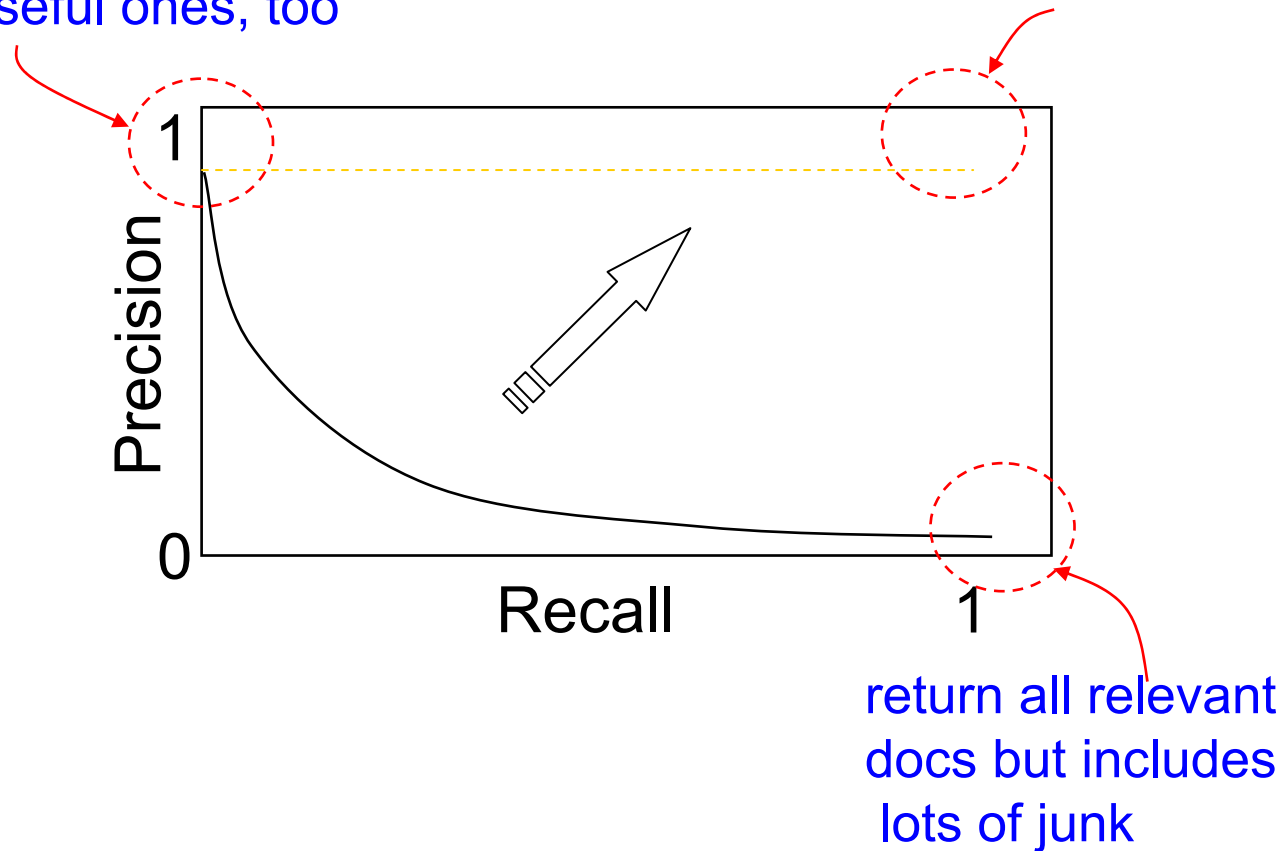
- Difficult to determine which of these two results is better

# Interpolated Recall-Precision Curve (cont.)

- Trade-off between Recall and Precision

return most relevant docs but miss many useful ones, too

the ideal case



# Interpolated Recall-Precision Curve (cont.)

- **Alternative**: average precision at a given document cutoff values (levels)
  - E.g.: compute the average precision when Top 5, 10, 15, 20, 30, 50 or 100 relevant documents have been seen
  - Focus on how well the system ranks the Top  $k$  documents
    - Provide additional information on the retrieval performance of the ranking algorithm
  - We can take (weighted) average over results

# Interpolated Recall-Precision Curve (cont.)

- Advantages
  - Simple, intuitive, and combined in single curve
  - Provide quantitative evaluation of the answer set and comparison among retrieval algorithms
  - *A standard evaluation strategy for IR systems*
- Disadvantages
  - Can't know true recall value except in small document collections (*document cutoff levels are needed!*)
  - Assume a strict document rank ordering



# Single Value Summaries

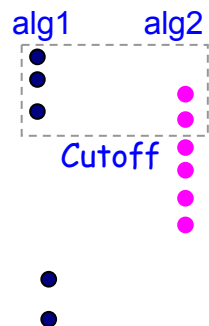
- Interpolated recall-precision curve
  - Compare the performance of retrieval algorithms over a set of example queries
    - Might disguise the important anomalies
  - How is the performance for each individual query ?
- A single precision value (for each query) is used instead
  - Interpreted as a summary of the corresponding precision versus recall curve
    - Just evaluate the precision based on the top 1 relevant document ?
    - Or averaged over all relevant documents

# Single Value Summaries (cont.)

- Method 1: **Average Precision** at Seen Relevant Documents
  - A single value summary of the ranking by averaging the precision figures obtained after each new relevant doc is observed

Example 3.2

1. $d_{123}$ • ( $P=1.0$ )	6. $d_9$ • ( $P=0.5$ )	11. $d_{38}$
2. $d_{84}$	7. $d_{511}$	12. $d_{48}$
3. $d_{56}$ • ( $P=0.66$ )	8. $d_{129}$	13. $d_{250}$
4. $d_6$	9. $d_{187}$	14. $d_{113}$
5. $d_8$	10. $d_{25}$ • ( $P=0.4$ )	15. $d_3$ • ( $P=0.3$ )
$(1.0+0.66+0.5+0.4+0.3)/5=0.57$		



– It favors systems which retrieve relevant docs quickly (early in the ranking)

– But when doc cutoff levels were used

- An algorithm might present a good average precision at seen relevant docs but have a poor performance in terms of overall recall

# Mean Average Precision (*mAP*)

- Averaged at relevant docs and across queries
  - E.g. relevant docs ranked at 1, 5, 10, precisions are 1/1, 2/5, 3/10,
    - non-interpolated average precision (or called *Average Precision at Seen Relevant Documents* in textbook)  
= $(1/1+2/5+3/10)/3$
  - Mean average Precision (*mAP*)

$$\frac{1}{|Q|} \sum_{q=1}^{|Q|} (\text{non - interpolated average precision})_q$$

- Widely used in IR performance evaluation

# Single Value Summaries (cont.)

- Method 2: R-Precision

- Generate a single value summary of ranking by computing the precision at the  $R$ -th position in the ranking
  - Where  $R$  is the **total number of relevant docs** for the current query

1. $d_{123}$ ●	6. $d_9$ ●	11. $d_{38}$
2. $d_{84}$	7. $d_{511}$	12. $d_{48}$
3. $d_{56}$ ● ■	8. $d_{129}$ ■	13. $d_{250}$
4. $d_6$	9. $d_{187}$	14. $d_{113}$
5. $d_8$	10. $d_{25}$ ●	15. $d_3$ ● ■

$R_q = \{d_3, d_5, d_9, d_{25}, d_{39}, d_{44}, d_{56}, d_{71}, d_{89}, d_{123}\}$   
• 10 relevant documents (●)  
=>  $R$ -precision =  $4/10 = 0.4$

$R_q = \{d_3, d_{56}, d_{129}\}$   
• 3 relevant document (■)  
=>  $R$ -precision =  $1/3 = 0.33$

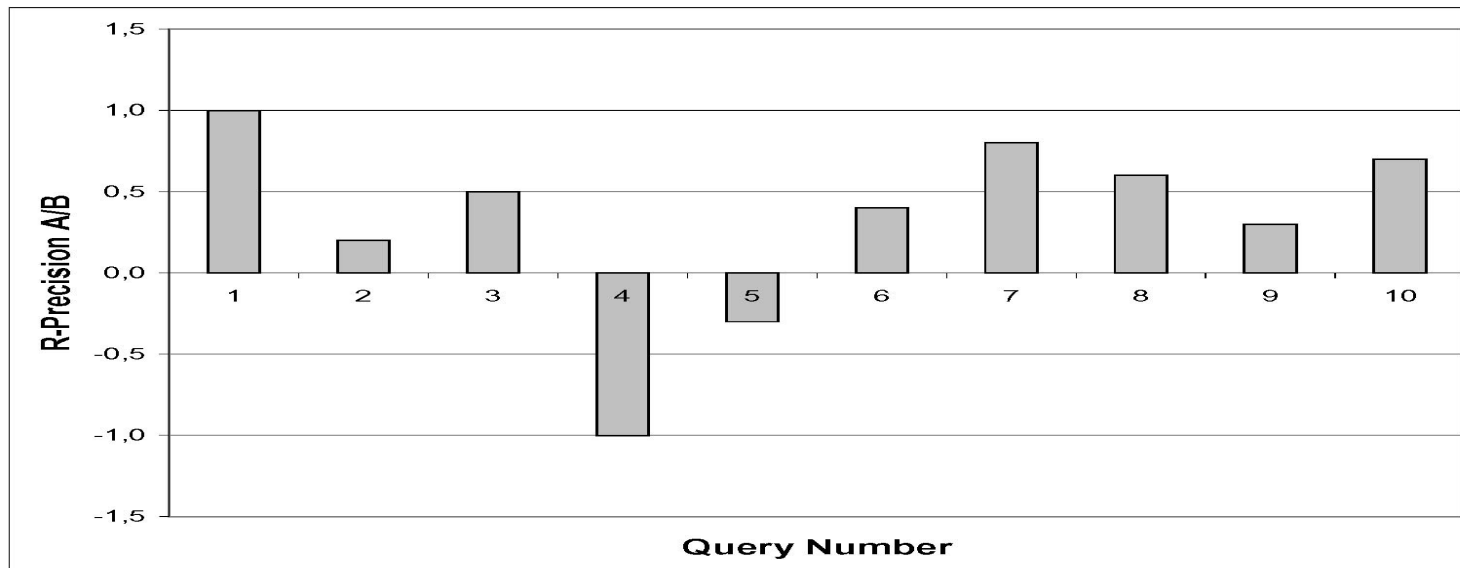
# Single Value Summaries (cont.)

- Method 3: Precision Histograms
  - Compare the retrieval history of two algorithms using the R-precision graph for several queries
    - A visual inspection
  - Example 3.5
    - Algorithms  $A$ ,  $B$
    - The difference of R-precision for the  $i$ -th query:

$$RP_{A/B}(i) = RP_A(i) - RP_B(i)$$

# Single Value Summaries (cont.)

- Method 3: Precision Histograms (cont.)
  - Example 3.5 (cont.)



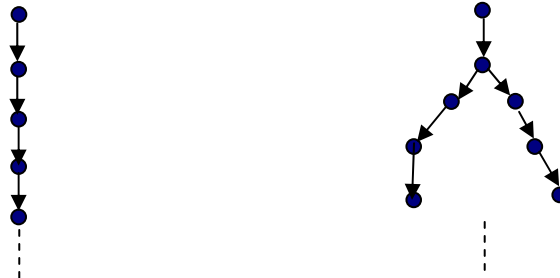
- A positive  $RP_{A/B}(i)$  indicates that the algorithm  $A$  is better than  $B$  for the  $i$ -th query and vice versa

# Single Value Summaries (cont.)

- Method 4: Summary Table Statistics
  - A statistical summary regarding the set of all the queries in a retrieval task
    - The number of queries used in the task
    - The total number of documents retrieved by all queries
    - The total number of relevant documents which were effectively retrieved when all queries are considered
    - The total number of relevant documents which could have been retrieved by all queries
    - ...

# Precision and Recall Appropriateness

- The proper estimation of maximal recall requires knowledge of all the documents in the collection
- Recall and precision are related measures which capture different aspects of the set of retrieved documents
- Recall and precision measure the effectiveness over queries in batch mode
- Recall and precision are defined under the enforcement of linear ordering of the retrieved documents
  - Partial Ordering ?





# Alternative Measures

- Method 1: The Harmonic Mean (F Measure)

- The harmonic mean  $F$  of recall and precision

$$F(j) = \frac{2}{\frac{1}{r(j)} + \frac{1}{P(j)}} = \frac{2 \cdot P(j) \cdot r(j)}{P(j) + r(j)}$$

- $r(j)$ : the recall for the  $j$ -th document in the ranking
- $P(j)$ : the precision for the  $j$ -th document in the ranking

- Characteristics

- $F = 0$ : no relevant documents were retrieved
- $F = 1$ : all ranked documents are relevant
- A high  $F$  achieved only when both recall and precision are high
- Determination of the maximal  $F$  can be interpreted as an attempt to find the best possible compromise between recall and precision

# Alternative Measures (cont.)

- Method 2: The E Measure

van Rijsbergen 1979

- Another measure which combines recall and precision
- Allow the user to specify whether he is more interested in recall or precision

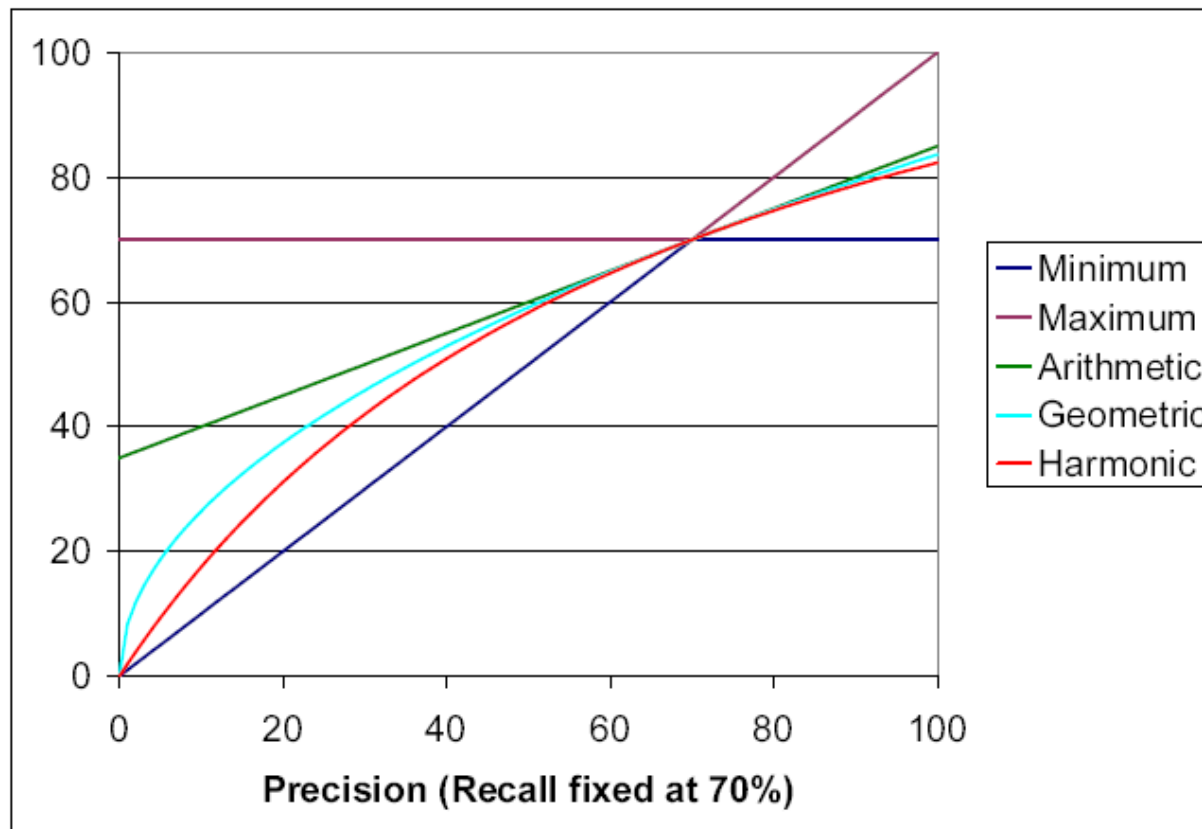
$$E(j) = 1 - \frac{1 + b^2}{\frac{b^2}{r(j)} + \frac{1}{P(j)}} = 1 - \frac{(1 + b^2) \cdot P(j) \cdot r(j)}{b^2 \cdot P(j) + r(j)}$$

- Characteristics

- $b = 1$ : act as the complement of F Measure
- $b > 1$ : more interested in recall
- $b < 1$ : more interested in precision

Wrong statements  
in the Textbook!

# Arithmetic/Geometric/Harmonic Means



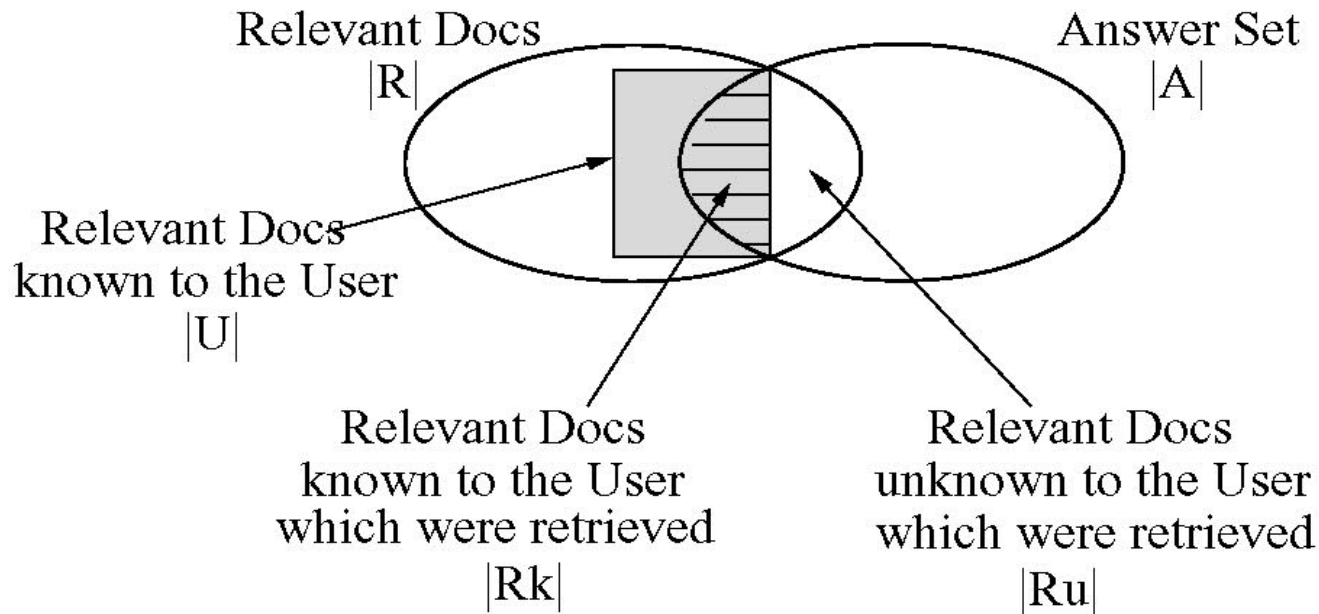
► **Figure 8.1** Graph comparing the harmonic mean to other means. The graph shows a slice through the calculation of various means of precision and recall for the fixed recall value of 70%. The harmonic means is always less than either the arithmetic or geometric mean, and often quite close to the minimum of the two numbers. When the precision is also 70%, all the measures coincide.

# Alternative Measures (cont.)

- Method 3: User-Oriented Measures
  - Problematic assumption of recall and precision
    - The set of relevant documents for a query is the same, independent of the user
  - However, different users have a different interpretation of document relevance
- User-oriented measures are therefore proposed
  - Coverage ratio
  - Novelty ratio
  - Relative recall
  - Recall effect

# Alternative Measures (cont.)

- Method 3: User-Oriented Measures (cont.)



- **Coverage ratio** =  $\frac{|R_k|}{|U|}$

- **Relative recall** =  $\frac{|R_k| + |R_u|}{|U|}$

- **Novelty ratio** =  $\frac{|R_u|}{|R_u| + |R_k|}$

- **Recall effect** =  $\frac{|U|}{|A|}$

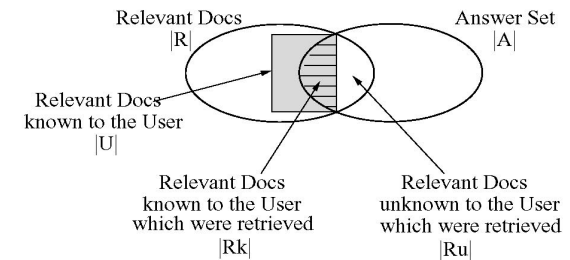
Measure the ability to reveal new relevant docs

# Alternative Measures (cont.)

- Coverage ratio

- The fraction of relevant docs **known** to the user which has been retrieved
- High → find most of the relevant docs user expected to see

$$\frac{|Rk|}{|U|}$$



- Novelty ratio

- The fraction of relevant docs retrieved which is **unknown** to the user
- High → find (reveal) many new relevant docs (information) the user previously unknown

$$\frac{|Ru|}{|Ru| + |Rk|}$$

# Alternative Measures (cont.)

- Relative recall

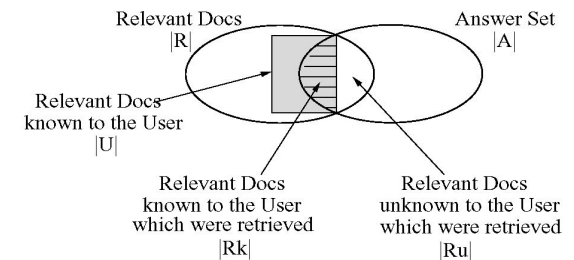
- The ratio between the number of relevant docs found by the system and the number of relevant docs the user expects to find

$$\frac{|R_k| + |R_u|}{|U|}$$

- Recall effect

- The ratio between the number of relevant docs the user expects to find and the number of docs found by the system

$$\frac{|U|}{|A|}$$



# Homework - 1

## Homework #1 :Evaluation Measures

The the query-document relevance information ([AssessmentTrainSet.txt](#)) for a set of queries (16 queries) and a collection of 2,265 documents is provided. An IR model is then tested on this query set and save the corresponding ranking results in a file ([ResultsTrainSet.txt](#)) . Please evaluate the overall model performance using the following two measures.

### 1. Interpolated Recall-Precision Curve:

$$\bar{P}_i(r_j) = \max_{r_j \leq r \leq r_{j-1}} P_i(r) \quad (\text{for each query})$$

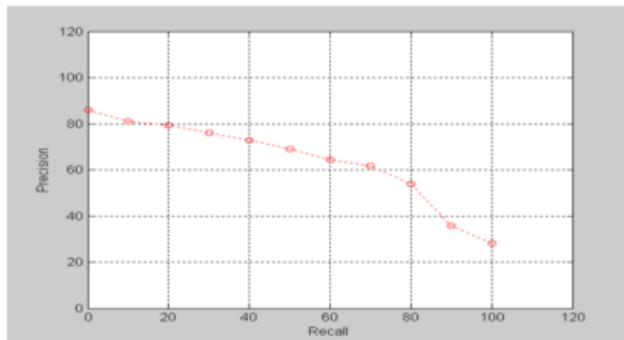
$$\bar{P}_{all}(r_j) = \frac{1}{N_q} \sum_{i=1}^{N_q} \bar{P}_i(r_j) \quad (\text{overall performance})$$

### 2. (Non-interpolated) Mean Average Precision:

$$\frac{1}{|Q|} \sum_{q=1}^{|Q|} (\text{non - interpolated average precision})_q$$

, where "non-interpolated average precision" is "average precision at seen relevant documents" introduced in the textbook.

### Example 1: Interpolated Recall-Precision Curve



### Example 2: (Non-interpolated) Mean Average Precision

**mAP=0.63787418**