# **Parsing with Context-Free Grammars**

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References:

1. Natural Language Understanding, Chapter 3 (3.1~3.4, 3.6)

2. Speech and Language Processing (1st ed.), Chapters 9, 10; (3rd ed.), Chapter 17

Grammars and Sentence Structures (1/3)

- Describe the structure of sentences and explore ways of characterizing all the legal structures in a language
	- **Parse tree**: how a sentence is broken into its major subparts (constituents), and how these subparts are broken up in turn
	- E.g., *John ate the cat*





Rewrite Rules



Grammar 3.2 A simple grammar

(NAME John))  $\mathsf{(\mathsf{S})}$  $(NP)$  $(V \text{ ate})$  $(VP)$ [list notation]  $(NP)$ (ART the)  $(N cat)$ )))

A set of rewrite rules describes what tree structures are allowable.

2 Formal models for capturing more sophisticated notions of grammatical structure, and algorithms for parsing these structures will be introduced.

Grammars and Sentence Structures (2/3)

- A grammar is said to derive a sentence if there is a sequence of rules that allow you to rewrite the start symbol S into the sentence
- A **top-down derivation strategy** starts with the **S** symbol and then searches through different ways to rewrite the symbols until the input sentence is generated

S

 $\Rightarrow$  NP VP (rewriting S)  $\Rightarrow$  NAME VP (rewriting NP) => John VP (rewriting NAME) => John V NP (rewriting VP)  $\Rightarrow$  John ate NP (rewriting V) => John ate ART N (rewriting NP)  $\Rightarrow$  John ate the N (rewriting ART)  $\Rightarrow$  John ate the cat (rewriting N) Grammars and Sentence Structures (3/3)

• In a **bottom-up derivation strategy**, you start with the words in the sentence and use the rewrite rules backward to reduce the sequence of symbols until it consists solely of S

> $\Rightarrow$  NAME ate the cat (rewriting John)  $\Rightarrow$  NAME Vthe cat (rewriting ate)  $\Rightarrow$  NAME V ART cat (rewriting the) => NAME V ART N (rewriting cat) => NP V ART N (rewriting NAME)  $\Rightarrow$  NP V NP (rewriting ART N) => NP VP (rewriting V NP)  $\Rightarrow$  S (rewriting NP VP)

- Ideal properties of a grammar: **generality**, **selectivity**, **understandability** 
	- **generality**, the range of sentences the grammar analyzes correctly; **selectivity**, the range of non-sentences it identifies as problematic; and **understandability**, the simplicity of the grammar itself

Recall: Context-Free Grammars (CFGs)

- Grammars consist entirely of rules with a single symbol on the left-hand side
- Formalized by Chomsky (1956), and Backus (1959)
	- Also called <mark>Backus-Naur Form (BNF)</mark>
- Also called phrase-structure grammars
- A CFG defines a formal language
- The most commonly used mathematical system for modeling the constituent structure in natural languages
	- **Ordering**
		- What are the rules that govern the ordering of words and bigger units in the language
	- **Constituency**
		- How do words group into units and what do we say about how the various kinds of units behave

https://en.wikipedia.org/wiki/Noam\_Chomsky 55



### Major Characteristics of CFGs (1/4)

- CFG examples
	- Consist of a set of **rules** (**productions**)

Rewrite the symbol on the left with a string of symbols on the right.



So starting from the symbol: **NP** we can use our first rule to rewrite **NP** as: **Det Nominal** and then rewrite **Nominal** as: **Noun** and finally rewrite these parts-of-speech as: **a flight**

When talking about these rules we can pronounce the right-arrow " *→ "* as "goes to", and so we might read the first rule above as "NP goes to Det Nominal".

Noun

flight

 $\overline{a}$ 

Major Characteristics of CFGs (2/4)

- Symbols used are divided into two classes: terminal and non-terminal symbols
	- A single **non-terminal symbols** on the left side of the arrow (→) while one or more terminal or non-terminal symbols on the right side
		- The **terminal symbol** is a word, while in **the lexicon**, the **non-terminal symbol** associated with each word is its lexical category, or part-of-speech (POS)
		- The **non-terminal symbol** can be a larger constituent (e.g. a phrasal unit) in addition to the lexical category

We can also represent a parse tree in a more compact format called **bracketed notation**:



Major Characteristics of CFGs (3/4)

- The notion of context in CFGs has nothing to do with the ordinary meaning of the **word context** in language
- All it really means that the non-terminal on the left-hand side of a rule is out of there all by itself

*A → B C*

• We can rewrite an *A* as a *B* followed by a *C* regardless of the context in which *A* is found

Major Characteristics of CFGs (4/4)

- **Generation** and **Parsing**: CFGs can be thought of as a device for generating sentences or a device for assigning a structure to a given sentence (i.e. parsing)
	- **Sentence generation**
		- Start from the *S* symbol, randomly choose and apply rewrite rules (or productions), until a sequence of words is generated

#### • **Parsing**

- Identify the structure of a sentence given a grammar
- Top-down or bottom-up strategies

More Complex Derivations (1/3)

- *S → NP VP*
	- Units *S*, *NP*, and *VP* are in the language
	- *S* consists of an *NP* followed immediately by a *VP*
	- There may be many kinds of S
	- NPs and VPs can occur at other places (on the left sides) of the set of rules

• E.g.

*NP → Det NominalNP <sup>→</sup> ProperNoum VP → Verb NPVP → Verb NP PP*

More Complex Derivations (2/3)



## More Complex Derivations (3/3)

- Recursion
	- The non-terminal on the left also appears somewhere on the right (directly or indirectly)

VP → VP PP [[departed Miami] [at noon]]

<u>NP</u> → <u>NP</u> PP [[The flight] [to Boston]]

- E.g.
	- flights <u>from Denver</u> (NP⇒NPPP)
	- Flights from Denver to Miami (NP PP⇒NP PP PP)
	- Flights from Denver to Miami <u>in February</u> (NP PP PP ⇒ NP PP PP PP)
	- Flights from Denver to Miami in February <u>on Friday (NP PP PP PP → NP PP PP PP</u> PP)

#### Formal Definition of Context-Free Grammar (1/3)

- A CFG *G* has four parameters ("4-tuple")
	- 1. A set of non-terminal symbols (or "variables")  $N$
	- 2. A set of terminal symbols  $\sum$  (disjoint from  $N)$
	- 3. A set of rules (productions)  $R$ , each of the form  $A \to \beta$ , where  $A$  is a non-terminal symbol and  $\beta$  is a string of symbols from the infinite set of strings  $(\sum\cup N)^*$
	- 4. A designated start symbol  $S$  (or  $N^{\rm 1})$
- CFG is a generative grammar
	- The language is defined by the set of possible sentences "**generated**" by the grammar
	- The concept of "**derivation**"
		- One string derives another one if it can be rewritten as the second one by **some series of rule applications**



Formal Definition of Context-Free Grammar (2/3)

- **Derivation**: a sequence of rules applied to a string that accounts for that string **NP** 
	- The whole process can be represented by a parse tree
	- E.g. a parse tree for "<mark>a flight</mark>"
- But, usually languages are derivable from the designated start symbol  $(S)$ 
	- The "sentence" node
	- The set of strings derivable from S called sentences



A derivation represented by a parse tree

#### Formal Definition of Context-Free Grammar (3/3)

#### • Directly Derive

 $A \to \beta$  is a production (rewrite) rule, where  $\alpha$  and  $\gamma$  are any strings in $(\sum\cup N)^*$ 

• Directly derive:  $A \to \beta \quad \alpha A \gamma \Rightarrow \alpha \beta \gamma$ 

where  $\alpha$ ,  $\beta$ ,  $r$ ,  $\alpha_i \in (\Sigma \cup N)$ ,  $m \geq 1$ 

• Derive 
$$
\alpha_1 \Rightarrow \alpha_2, \alpha_2 \Rightarrow \alpha_3, ..., \alpha_{m-1} \Rightarrow \alpha_m \therefore \alpha_1 \stackrel{*}{\Rightarrow} \alpha_m
$$

 $(\alpha_1$  derives  $\alpha$  $\alpha_1, \alpha_2, ...$  ,  $\alpha_m$  are any strings in(∑ ∪ N) $^*$   $(\alpha_1$  derives  $\alpha_m)$ 

• **Syntactic parsing:** the problem of mapping from a string of words to its parse tree(s) is called syntactic parsing

#### **Treebanks**

- A corpus in which every sentence is annotated with a parse tree is called a **treebank**
- Treebanks play an important role in parsing as well as in linguistic investigations of syntactic phenomena
- Treebanks are generally made by parsing each sentence with a parse that is then hand-corrected by human linguists
- The Penn Treebank project constructed various treebanks in English, Arabic, and **Chinese**





CFG: Conversion to Chomsky Normal Form (1/2)

• A context-free grammar is in Chomsky normal form (CNF) (Chomsky, 1963) if it is  $\epsilon$ -free and if in addition each production is either of the form

 $A \rightarrow B C$ or,  $A \rightarrow a$ 

- For CNF, the right-hand side of each rule either has two non-terminal symbols or one terminal symbol
- Chomsky binary normal form grammars are binary branching, that is they have binary trees (down branching to the **prelexical nodes** )

CFG: Conversion to Chomsky Normal Form (2/2)

• Any context-free grammar (CFG) can be converted into a weakly equivalent Chomsky normal form (CNF) grammar



Two grammars **are weakly equivalent** if they generate the same set of strings but do not assign the same phrase structure to each sentence.

## Ambiguity (1/2)

- **Structural ambiguity** (viz. assigning more than one parse to a sentence) is the most serious problem faced by syntactic parsers
	- Two common kinds of ambiguity are **attachment ambiguity** and **coordination ambiguity**
	- A sentence has an attachment ambiguity if a particular constituent can be attached to the parse tree at more than one place.



#### **attachment ambiguity:**

**coordination ambiguity: [old [men and women]] [old men] and [women]**

**Figure 17.9** Two parse trees for an ambiguous sentence. The parse on the left corresponds to the humorous reading in which the elephant is in the paiamas, the parse on the right corresponds to the reading in which Captain Spaulding did the shooting in his pajamas.

## Ambiguity (2/2)

• Various kinds of **adverbial phrases** are also subject to the attachment ambiguity, for example:

We saw the Eiffel Tower flying to Paris.

• The gerundive-VP flying to Paris can be part of a gerundive sentence whose subject is the Eiffel Tower or it can be an adjunct modifying the VP headed by saw

#### Lexical Head

- Syntactic constituents can be associated with a **lexical head**
	- For example, N is the head of an NP, V is the head of a VP
- In one simple model of lexical heads, each context-free rule is associated with a head
- The head is the word in the phrase that is grammatically the most important
- Heads are passed up the parse tree; thus, each non-terminal in a parse tree is annotated with a single word, which is its lexical head



Each CFG rule must be augmented to identify one right-side constituent to be the head child.

Sentence–level Construction of English

• **Declaratives**: A plane left.

S *→* NP VP

• **Imperatives**: Show the lowest fare.

S *→* VP

• **Yes-No Questions**: Did the plane leave?

S *→* Aux NP VP

• **WH Questions**: When did the plane leave?

S *→* WH Aux NP VP

Parsing Strategies (1/2)

- Top-Down Parsing
	- Start with the *S* symbol and search through different ways to rewrite the symbols until the input sentence is generated, or until all possibilities have been explored
- Bottom-Up Parsing
	- Start with the words in the input sentence and use the rewrite rules backward to reduce the sequence of symbols until it consists solely of *S*
	- The left side of each rule is used to rewrite the symbols on the right side
		- Take a sequence of symbols and match it to the right side of the rule

## Parsing Strategies (2/2)

#### **Parsing as Search**

• Different search algorithms, such as depth-first search (DFS) or breadth-first search (BFS) algorithms, can be applied



The record of the parsing process, either in top‐down or bottom‐up manners, can be used to generate the parse tree representation.

The Top-Down Parser

- Start with the *S* symbol and rewrite it into a sequence of terminal symbols that matches the classes of the words in the input sentence
	- The state of the parse at any given time can be represented as a list of symbols that are the results of operations applied so far



#### The Simple Top-Down Parser (1/3)



**Figure 3.5** Top-down depth-first parse of  $_1$  The  $_2$  dogs  $_3$  cried  $_4$ 



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## The Simple Top-Down Parser (2/3)

• Algorithm

**Step 1:** Select the current state: take the first state off the possibilities list and call it **C**

- If the possibilities list is empty, then the algorithm fails
- **Step 2:** If **C** consists of an empty symbol list and is at the sentence end position, the algorithm succeeds

**Step 3:** Otherwise, generate the next possible states

• If the first symbol on the symbol list is a <mark>lexical symbol (part-of-speech tag</mark>), and the next word in the sentence can be (matched) in that class, then create a new state by removing the first symbol from the symbol list and update the word position, and **add it to the possibilities list** 

where to pu<sup>t</sup> the new states depends on the searchstrategies (e.g., BFS or DFS)

• Otherwise, if the first symbol on the symbol list of **C** is a non-terminal (but not the lexical symbol), generate a new state for each rule in the grammar that can rewrite that non-terminal symbol and **add them all to the possibilities list** 

The Simple Top-Down Parser (3/3)



**Figure 3.6** A top-down parse of  $_1$  The  $_2$  old  $_3$  man  $_4$  cried  $_5$ 

**the possibilities list** 

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Search strategies for the Top-Down Parser (1/3)

- Depth-first search: DFS (LIFO: last-in first-out)
	- The possibilities list is a stack
	- Step 1 always take the first element off the list
	- Step 3 always puts (adds) the new states **on the front of the list**
- Breadth-first search: BFS (FIFO: first-in first-out)
	- The possibilities list is a queue
	- Step 1 always take the first element off the list
	- Step 3 always puts (adds) the new states **on the end of the list**

Search strategies for the Top-Down Parser (2/3)



Figure 3.7 Search tree for two parse strategies (depth-first strategy on left; breadth-first on right)

### Search strategies for the Top-Down Parser (3/3)

- Comparison of DFS and BFS
	- DFS
		- **One interpretation** is considered and expanded until fails; only then is the second one considered
		- Often moves quickly to the a solution but in other cases may spend considerable time pursuing futile paths
	- BFS
		- **All interpretations** are considered alternatively, each being expanded one step at a time
		- Explore each possible solution to a certain depth before moving on

Many parsers built today use the DFS strategy because it tends to minimize the no. of backup states needed and thus uses less memory and requires less bookkeeping.

The Bottom-Up Parser (1/2)

- Start with the words of the input, and try to build tree from the words up, by applying rules from the grammar one at a time
	- The right hand side of some rules might fit
	- Successful if the parser succeeds in building a tree rooted in the start symbol (or a symbol list with S and positioned at the end of the input sentence) that covers all the input

#### The Bottom-Up Parser (2/2)



1 **Book** 2 **that** 3 **flight** 4



## Comparing Top-Down and Bottom-UP Parsing

#### • Top-Down

- Pro: Never wastes time exploring trees that cannot result in an *S*
- Con: But spends considerable effort on *S* trees that are not consistent with the input
- Bottom-Up
	- Pro: Never suggest trees that are not least locally grounded in the actual input
	- Con: Trees that have no hope of leading to an S, or fitting in with any of their neighbors, are generated with wild abandon
	- Pro: Only check the input once

Problems with Parsing (1/4)

#### • **Left-recursion**

• A non-terminal category that has a derivation that includes itself anywhere along its leftmost branch

 $\mathsf{NP} \rightarrow \mathsf{Det}$  Nominal

Det  $\rightarrow$  NP 's

• Especially, **the immediately** left-recursive rule

 $\mathsf{NP} \to \mathsf{NP}$  's  $\mathsf{N}$ 

• E.g. causing a infinite loop in top-down parsing with DFS search strategy



Problems with Parsing (2/4)

#### • **Ambiguity**

- Structural ambiguity: arises in the syntactic structures used in parsing
	- The grammar assigns more than one possible parse to a sentence
		- Attachment ambiguity: Most frequently seen for adverbial phrases (PP-attachment ambiguity)

I shot an elephant in my pajamas.

• Coordination ambiguity

old men and women

Parsers which do not incorporate disambiguators must simply return all the possible parse trees for a given input.
Problems with Parsing (3/4)

#### • **Ambiguity**

Basic ways to alleviate the ambiguity problem

- Dynamic programming
	- Used to exploit the regularities in the search space so that the common subpart are derived only once
		- Reduce some of the costs associated with ambiguity
	- Implicitly store all possible parses by storing all the constituents with links that enable the parses to be reconstructed
- Heuristic search
	- Augment the parser's search strategy with heuristics that guide it towards likely parses first

Problems with Parsing (4/4)

- Repeated Parsing of Subtrees
	- The parser often builds valid subtrees for portions of the input, then discards them during the backtracking
		- It has to rebuild these subtrees again
		- $\bullet$ Some constituents are constructed more than once



spoon  $\mathfrak a$ 

An example of a prepositional phrase attachment ambiguity. The same state of the state of  $^{38}$ 

# CKY Parsing (1/2)

- The **dynamic programming** advantage arises from the context-free nature of our grammar rules
	- Once a constituent has been discovered in a segment of the input we can record its presence and make it available for use in any subsequent derivation that might require it
- The Cocke-Kasami-Younger (CKY) algorithm is the most widely used dynamic-programming based approach to parsing
	- **Chart parsing** (Kaplan 1973, Kay 1982) is a related approach
	- Dynamic programming methods are often referred to as chart parsing methods

CKY Parsing (2/2)

- The CKY algorithm requires grammars to first be in Chomsky Normal Form (CNF), namely being binary branching
	- Recall that CNF: A  $\rightarrow$  B C or A  $\rightarrow$  w
- The entire conversion process can be summarized as follows:
	- 1. Copy all conforming rules to the new grammar unchanged
	- 2. Convert terminals within rules to dummy nonterminals $INF-VP \rightarrow TO VP$

 $TO \rightarrow to$ 

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INF-VP \rightarrow to VP
```
- 3. Convert unit productions
- 4. Make all rules binary and add them to new grammar <sup>40</sup>



**Figure 17.10**  $\mathcal{L}_1$  Grammar and its conversion to CNF. Note that although they aren't shown here, all the original lexical entries from  $\mathcal{L}_1$  carry over unchanged as well.

Nominal  $\rightarrow$  Noun  $\longrightarrow$  Nominal  $\rightarrow$  book | flight | meal | ...

## CYK Recognition (1/4)

• Employ the CYK algorithm to tell whether a valid exists for a given sentence based on whether or not CYK finds an S in cell [0, *n*] of the **parse table** we maintain for a sentence of length *<sup>n</sup>*



 $\mathsf{Book}_1$  the $_2$  flight $_3$  through $_4$  Houston $_5$ 

Completed parse table for *Book the flight through Houston*.

Speech and Language Processing (3rd ed.), Chapter 17

## CYK Recognition (2/4)

• The CKY algorithm

Complexity: O(N3L)?



- The outermost loop of the algorithm iterates over the columns, from left to right
- •The second loop iterates over the rows, from the bottom up
	- At each such split *k*, the algorithm considers whether the contents of the two cells can be combined in a way that is sanctioned by (被認可) a rule in the grammar



CYK Recognition (4/4)

• This figure shows how the five cells of column 5 of the table are filled after the word Houston is read

 $\mathsf{Book}_1$  the $_2$  flight $_3$  through $_4$  Houston $_5$ 



## Change CKY Recognition to CKY Parsing

- To turn **CKY recognition** into a **parser** capable of returning all possible parses for a given input, we can make two simple changes to the algorithm
	- 1. The first change is to augment the entries in the table so that each non-terminal is paired with pointers to the table entries from which it was derived
	- 2. The second change is to permit multiple versions of the same non-terminal to be entered into the table

## Span-Based Neural Constituency Parsing (1/4)

- **Neural CKY** (Kitaev et al., 2018, 2019)
	- Train a neural classifier to assign a score to each constituent
		- Introduce a parser that combines an **encoder** built using this kind of self-attentive architecture with a **decoder** customized for **chart parsing**

Berkeley

- Then, use a modified version of CKY to combine these constituent scores to find the best-scoring parse tree
- More properties of **Neural CKY**
	- Learns to map a span of words to a constituent, and, like CKY, hierarchically combines larger and larger spans to build the parse-tree bottom-up
	- But unlike CKY, this parser does not use the hand-written grammar to constrain what constituents can be combined, instead just relying on the learned neural representations of spans to encode likely combinations

2. N. Kitaev et al.,"Multilingual constituency parsing with self‐attention and pre‐training ,"ACL 2019

<sup>1.</sup> N. Kitaev and D. Klein, "Constituency parsing with a self-attentive encoder," ACL 2018

## Span-Based Neural Constituency Parsing (2/4)



A simplified outline of computing the span score for the span the flight with **Figure 17.15** the label NP.

## Span-Based Neural Constituency Parsing (3/4)

- The output vector of each word  $y_t$  is spilt into two halves  $(\bar y_t; \bar y_t)$ 
	- A (leftward-pointing) vector for spans ending at this fencepost (護欄柱),  $\tilde{\mathrm{y}}_t$ , and a (rightward-pointing) vector  $\vec{y}_t$  for spans beginning at this fencepost
	- More specifically, even coordinates contribute to  $\bar{\mathrm{y}}_t$  and old coordinates contribute to  $\bar{\mathrm{y}}_t$



#### Span-Based Neural Constituency Parsing (4/4)

• Choose the final parse tree that has the maximum score

$$
\hat{T} = \frac{\text{argmax}}{T} s(T)
$$

- A variant of the CYK algorithm (Dynamic Programming)
	- For spans of length 1

$$
s_{\text{best}} = \frac{\max}{l} s(i, i+1, l)
$$

• For other spans (of length >1)

$$
s_{\text{best}}(i,j) = \frac{\max_l s(i,j,l) + \frac{\max_l s_{\text{best}}(i,k) + s_{\text{best}}(k,j)]}{k}
$$

 $\Delta$ 9 This discriminative parser achieved a state‐of‐the‐art result (with an F1 score of 0.9355) on the PennTreebank dataset at that time.

## The Bottom-Up Chart Parser (1/29)

- A data structure called chart is introduced
	- Allow the parser to store the partial results of matching as it done so far
	- Such that the work would not be reduplicated
- The basic operation of a chart parser involves combining an active arc with a complete constituents (keys)
	- Three kinds of data structures
		- The agenda (to store new complete constituents)
		- The active arcs (i.e. partial parse trees)
		- The chart



Space Complexity vs. Time Complexity 50 and 50 and 50 and 50 separate  $50$ 

- A subtree corresponding to
- a single grammar rule
- Information about the progress made in completing the subtree
- Position of the subtree has to do with the input

```
The Bottom-Up Chart Parser (2/29)
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- The Bottom-Up Chart Parsing Algorithm
	- 1. If the agenda is empty, look up the **interpretations** for the next word in the input and add them to the agenda
	- 2. Select a constituent from the agenda (Call it constituent C from position  $\bm{{\mathsf{p}}}_1$  to  $\bm{{\mathsf{p}}}_2$ )
	- 3. For each rule in the grammar of form Ⅹ →CX<sub>1</sub>...X<sub>n</sub> , add **an active arc** of form  $\mathsf{X} \to^\circ \mathsf{CX}_1 ... \mathsf{X_n}$  from position  $\mathsf{p}_1$  to  $\mathsf{p}_1$
	- 4. Add C to the chart using the following **arc extension algorithm**

4.1 Insert C **into the chart** from position  $\mathsf{p}_\mathtt{1}$  to  $\mathsf{p}_\mathtt{2}$ 

- 4.2 For any active arc of the form  $X \rightarrow X_1... \circ C...X_n$  from position  $\bm{{\mathsf{p}}}_0$  to  $\bm{{\mathsf{p}}}_1$  add a new active arc  $\bm{\mathsf{X}}\!\rightarrow\!\!\bm{\mathsf{X}}_1...$  C  $^\circ$  … $\bm{\mathsf{X}}_\textsf{n}$  from  $\bm{{\mathsf{p}}}_0$  to  $\bm{{\mathsf{p}}}_2$
- 4.3 For any active arc of the form  $X \rightarrow X_1... X_n \circ C$  from position

 $\bm{{\mathsf{p}}}_0$  to  $\bm{{\mathsf{p}}}_1$  then add a new constituent of type X from  $\bm{{\mathsf{p}}}_0$  to  $\bm{{\mathsf{p}}}_2$ 

**to the agenda** 

Loop until no input left

#### The Bottom-Up Chart Parser (3/29)

#### • Example



#### **Initialization**



Input:  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 

> Note that depth-first strategy is used here => The agenda is implemented as a stack.

# The Bottom-Up Chart Parser (4/29)

#### • Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 1**

**Enter ART1: (***the* **from 1 to 2 )** Look at next word



1. If the agenda is empty, look up the interpretations for the next word in the input and add them to the agenda 2. Select a constituent from the agenda (Call it constituent C from position  $p_1$  to  $p_2$ ) 3. For each rule in the grammar of form  $X \rightarrow CX_1...X_n$ , add an active arc of form  $X \rightarrow \in C X_1...X_n$  from position  $p_1$  to  $p_1$ 4. Add C to the chart using the following arc extension algorithm 4.1 Insert C into the chart from position  $p_1$  to  $p_2$ 4.2 For any active arc of the form  $X \rightarrow X_1... \cdot C...X_n$  from position  $p_0$  to  $p_1$  add a new active arc  $X \rightarrow X_1...$  C  $\cdot ...X_n$  from  $p_0$  to  $p_2$ 

- 4.3 For any active arc of the form  $X \rightarrow X_1... X_n \cdot C$  from position  $p_0$  to  $p_1$  then add a new constituent of type X from  $p_0$  to  $p_2$ 
	- to the agenda

• The Bottom-Up Chart Parsing Algorithm

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### The Bottom-Up Chart Parser (5/29)

• Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



to the agenda the: ARTlarge: ADJ can: N, AUX hold: N, V Water: N

#### **Loop 1**



• The Bottom-Up Chart Parsing Algorithm 1. If the agenda is empty, look up the interpretations for the next word in the input and add them to the agenda 2. Select a constituent from the agenda (Call it constituent C from position  $p_1$  to  $p_2$ ) 3. For each rule in the grammar of form  $X \rightarrow CX_1...X_n$ , add an active arc of form  $X \rightarrow \in C X_1...X_n$  from position  $p_1$  to  $p_1$ 4. Add C to the chart using the following arc extension algorithm 4.1 Insert C into the chart from position  $p_1$  to  $p_2$ 4.2 For any active arc of the form  $X \rightarrow X_1... \cdot C...X_n$  from position  $p_0$  to  $p_1$  add a new active arc  $X \rightarrow X_1...$  C  $\cdot ...X_n$  from  $p_0$  to  $p_2$ 4.3 For any active arc of the form  $X \rightarrow X_1... X_n \cdot C$  from position  $p_0$  to  $p_1$  then add a new constituent of type X from  $p_0$  to  $p_2$ 

## The Bottom-Up Chart Parser (6/29)

• Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $\overline{\phantom{a}}$ 



• The Bottom-Up Chart Parsing Algorithm 1. If the agenda is empty, look up the interpretations for the next word in the input and add them to the agenda 2. Select a constituent from the agenda (Call it constituent C from position  $p_1$  to  $p_2$ ) 3. For each rule in the grammar of form  $X \rightarrow CX_1...X_n$ , add an active arc of form  $X \rightarrow \in C X_1...X_n$  from position  $p_1$  to  $p_1$ 4. Add C to the chart using the following arc extension algorithm 4.1 Insert C into the chart from position  $p_1$  to  $p_2$ 4.2 For any active arc of the form  $X \rightarrow X_1... \cdot C...X_n$  from position  $p_0$  to  $p_1$  add a new active arc  $X \rightarrow X_1...$  C  $\cdot ...X_n$  from  $p_0$  to  $p_2$ 4.3 For any active arc of the form  $X \rightarrow X_1... X_n \circ C$  from position 7 $p_0$  to  $p_1$  then add a new constituent of type X from  $p_0$  to  $p_2$ to the agenda the: ARTlarge: ADJ

can: N, AUX hold: N, V Water: N

**Loop 2** Look at next word

**Enter ADJ1: ("***large"* **from 2 to 3)**



#### The Bottom-Up Chart Parser (7/29)

#### • Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



the: ARTlarge: ADJ can: N, AUX hold: N, V Water: N

#### **Loop 2** ( using the arc extension algorithm)

#### **Enter ADJ1: ("***large"* **from 2 to 3)**





4.2 For any active arc of the form  $X \rightarrow X_1... \cdot C...X_n$  from position  $p_0$  to  $p_1$  add a new active arc  $X \rightarrow X_1...$  C  $\cdot ...X_n$  from  $p_0$  to  $p_2$ 

4.3 For any active arc of the form  $X \rightarrow X_1... X_n \circ C$  from position  $p_0$  to  $p_1$  then add a new constituent of type X from  $p_0$  to  $p_2$ 

to the agenda

#### The Bottom-Up Chart Parser (8/29)

• Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



large: ADJ can: N, AUX hold: N, V Water: N

• The Bottom-Up Chart Parsing Algorithm

 $X \rightarrow \in C X_1...X_n$  from position  $p_1$  to  $p_1$ 

and add them to the agenda

to the agenda

1. If the agenda is empty, look up the interpretations for the next word in the input

2. Select a constituent from the agenda (Call it constituent C from position  $p_1$  to  $p_2$ ) 3. For each rule in the grammar of form  $X \rightarrow CX_1...X_n$ , add an active arc of form

position  $p_0$  to  $p_1$  add a new active arc  $X \rightarrow X_1...$  C  $\cdot ...X_n$  from  $p_0$  to  $p_2$ 

4. Add C to the chart using the following arc extension algorithm 4.1 Insert C into the chart from position  $p_1$  to  $p_2$ 4.2 For any active arc of the form  $X \rightarrow X_1... \cdot C...X_n$  from

4.3 For any active arc of the form  $X \rightarrow X_1... X_n \cdot C$  from position  $p_0$  to  $p_1$  then add a new constituent of type X from  $p_0$  to  $p_2$ 

**Loop 3** Look at next word

**Enter N1: (***"can"* **from 3 to 4)**



The Bottom-Up Chart Parser (9/29)

• Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 3**

**Enter N1: (***"can"* **from 3 to 4)** ( using the arc extension algorithm)



## The Bottom-Up Chart Parser (10/29)

# • Example  $\qquad \qquad \frac{1}{1}$

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 4**

**Enter NP1: ("***the* **large can" from 1 to 4)**



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### The Bottom-Up Chart Parser (11/29)

#### • Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 4**

**Enter NP1: ("***the* **large can" from 1 to 4)** ( using the arc extension algorithm)



#### The Bottom-Up Chart Parser (12/29)

#### • Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 1. S  $\rightarrow$  NP VP 2. NP  $\rightarrow$  ART ADJ  $\,$  N 3. NP  $\rightarrow$  ART N 4. NP  $\rightarrow$  ADJ N 5. VP  $\rightarrow$  AUX VP 6. VP  $\rightarrow$  V NP  $^+$ the: ART large: ADJ can: N, AUX hold: N, V Water: N

**Loop 5**

**Enter NP2: (***"large can"* **from 2 to 4)**



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#### The Bottom-Up Chart Parser (13/29)

#### • Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 

N, AUX



**Loop 5** ( using the arc extension algorithm)

**Enter NP2: (***"large can"* **from 2 to 4)**



## The Bottom-Up Chart Parser (14/29)

#### • Example



#### The Bottom-Up Chart Parser (15/29)

•Example  $1$  The

#### $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$



**Loop 6** ( using the arc extension algorithm)

#### **Enter AUX1: (***"can"* **from 3 to 4)**



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### The Bottom-Up Chart Parser (16/29)

• Example  $\qquad \qquad \overline{\phantom{a}}_{\phantom{a}1}$ 

#### $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 1.  $S \rightarrow NP VP$ 4.  $NP \rightarrow ADJ N$ the: ARTlarge: ADJ

5. VP  $\rightarrow$  AUX VP 6. VP  $\rightarrow$  V NP  $\overline{ }$ 

can: N, AUX hold: N, V Water: N

**Loop 7** Look at next word

2. NP  $\rightarrow$  ART ADJ N

3. NP  $\rightarrow$  ART N

**Enter N2: (***"hold"* **from 4 to 5)**



## The Bottom-Up Chart Parser (17/29)

• Example

#### $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$



**Loop 7** ( using the arc extension algorithm)

**Enter N2: (***"hold"* **from 4 to 5)**



## The Bottom-Up Chart Parser (18/29)



**Enter V1: (***"hold"* **from 4 to 5)**



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#### The Bottom-Up Chart Parser (19/29)

• Example

#### $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$



**Loop 8** ( using the arc extension algorithm)

**Enter V1: (***"hold"* **from 4 to 5)**



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## The Bottom-Up Chart Parser (20/29)



**Enter ART2: ("***the"* **from 5 to 6)**



#### The Bottom-Up Chart Parser (21/29)

- Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 1.  $S \rightarrow NP VP$ 2. NP  $\rightarrow$  ART ADJ  $\,$  N 3. NP  $\rightarrow$  ART N 4.  $NP \rightarrow ADJ$  N 5. VP  $\rightarrow$  AUX VP 6. VP  $\rightarrow$  V NP  $\overline{ }$ the: ART large: ADJ can: N, AUX hold: N, V Water: N
	- **Loop 9** ( using the arc extension algorithm)

**Enter ART2: ("***the"* **from 5 to 6)**



## The Bottom-Up Chart Parser (22/29)



 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 

• Example



**Enter N3: (***"water"* **from 6 to 7)**



#### The Bottom-Up Chart Parser (23/29)


#### The Bottom-Up Chart Parser (24/29)



#### The Bottom-Up Chart Parser (25/29)



## The Bottom-Up Chart Parser (26/29)

•



**Enter VP1: (***"hold the water"* **from 4 to 7)**



The Bottom-Up Chart Parser (27/29)

• Example



# The Bottom-Up Chart Parser (28/29)



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#### The final Chart

• Example



Since you have derived an S covering the entire sentence, you can stop successfully. If you wanted to find all possible interpretations for the sentence, you would continue parsing until the agenda became empty.

# The Bottom-Up Chart Parser (29/29)

- Characteristics
	- The algorithm always moves forward through the chart making additions as it goes
	- Arcs are never removed and the algorithm never backtracks to a previous chart entry once it has moved on
	- Different *S* structures might share the common subparts represented in the chart only once

# The Top-Down Chart Parser (1/27)

• The Top-Down Chart Parsing Algorithm

**Initialization**: For every rule in the grammar of form S  $\rightarrow$ X $_1...$ X $_k$  , add an arc labeled S  $\rightarrow$   $\stackrel{\circ}{\sim}$   $\mathsf{X}_1...\mathsf{X}_\mathsf{k}$  using **the arc introduction algorithm** 

- 1. If the agenda is empty, look up the interpretations of the next word and add them to the agenda
- 2. Select a constituent C from the agenda
- 3. Using **the arc extension algorithm**, combine C with every active arc on the chart. Any new constituents are added to the agenda
- 4. For any active arcs created in step 3, add them to the chart using the following **top-down arc introduction algorithm**
	- To add an arc S  $\rightarrow$  C $_{1}...$  。C $_{\sf i}...$ C $_{\sf n}$  ending at position j, do the following:

Loop until no input left

For each rule in the grammar of form  ${\sf C_i}\to{\sf X_{1}}...{\sf X_{k}},\;$  recursively add the new arc  $\text{C} _{\text{i}} \rightarrow \text{ }^{\text{o}} \text{ X} _{\text{1}} ... \text{ X} _{\text{k}}$  from position j to j

The Top-Down Chart Parser (2/27)

#### • Recall "**the arc extension algorithm** "

- Insert C **into the chart** from position  $\mathsf{p}_1$  to  $\mathsf{p}_2$
- For any active arc of the form X  $\to$ X $_1...$   $^{\circ}$  C…X $_{\mathsf{n}}$  from position  $\mathsf{p}_0$  to  $\mathsf{p}_1$  add a new active arc X  ${\rightarrow}$ X $_1...$  C  $^{\circ}$  ...X $_{\mathsf{n}}$  from  $\mathsf{p}_{\mathsf{0}}$  to  $\mathsf{p}_{\mathsf{2}}$
- For any active arc of the form  ${\mathsf X}\to {\mathsf X}_1...$   ${\mathsf X}_{\mathsf n}$   $\circ$  C from position  ${\mathsf p}_0$  to  ${\mathsf p}_1$  then add a new constituent of type X from p 0 to p2 **to the agenda**

## The Top-Down Chart Parser (3/27)

• Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Initialization:** ( using the arc introduction algorithm)



The Top-Down Chart Parser (4/27)

• Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



# The Top-Down Chart Parser (5/27)

#### • Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 1**

**Enter ART1 (***"the"* **from 1 to 2 ):**  ( using the arc extension algorithm)



The Top-Down Chart Parser (6/27)

• Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 2**

**Enter ART1 (***"large"* **from 2 to 3):** Look at next word



## The Top-Down Chart Parser (7/27)

• Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 2**

**Enter ADJ1 (***"large"* **from 2 to 3):** ( using the arc extension algorithm)



The Top-Down Chart Parser (8/27)

• Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 3**

**Enter N1 (***"can"* **from 3 to 4):** Look at next word



## The Top-Down Chart Parser (9/27)

• Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 

1. S  $\rightarrow$  NP VP

3. NP  $\rightarrow$  ART N

2. NP  $\rightarrow$  ART ADJ  $\,$  N

the: ART large: ADJ can: N, AUX hold: N, V Water: N

**Loop 3**

**Enter N1 (***"can"* **from 3 to 4):** ( using the arc extension algorithm)

4. NP  $\rightarrow$  ADJ N 5. VP  $\rightarrow$  AUX VP 6. VP  $\rightarrow$  V NP  $^+$ 



# The Top-Down Chart Parser (10/27)

• Example

#### $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$



Water: N

**Loop 4**

**Enter NP1 (***"the large can"* **from 1 to 4):**



### The Top-Down Chart Parser (11/27)

• Example

#### $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$



**Loop 4**

#### **Enter NP1 (***"the large can"* **from 1 to 4):**

( using the arc extension algorithm)



### The Top-Down Chart Parser (12/27)

• Example

#### $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$



**Loop 5**

**Enter AUX1 (***"can"* **from 3 to 4):**



The Top-Down Chart Parser (13/27)

 ${\sf NP} \to$   $\,{\scriptstyle \circ }$  ADJ  $\,{\sf N}$ 

#### • Example



The Top-Down Chart Parser (14/27)

#### • Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 6**

**Enter N2 (***"holds"* **from 4 to 5):** Look at next word



The Top-Down Chart Parser (15/27)

• Example

 $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



## The Top-Down Chart Parser (16/27)

• Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



the: ART

Water: N

**Loop 7**

**Enter V1 (***"holds"* **from 4 to 5):**



#### The Top-Down Chart Parser (17/27)

• Example

1.  $S \rightarrow NP VP$ 2. NP  $\rightarrow$  ART ADJ  $\,$  N 3. NP  $\rightarrow$  ART N 4. NP  $\rightarrow$  ADJ N 5. VP  $\rightarrow$  AUX VP 6. VP  $\rightarrow$  V NP  $\overline{ }$  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ the: ARTlarge: ADJ can: N, AUX hold: N, V Water: N

**Loop 7**

```
Enter V1 ("holds" from 4 to 5):
```
( using the arc extension algorithm)



## The Top-Down Chart Parser (18/27)

#### • Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$



**Loop 8**

**Enter ART2 (***"the"* **from 5 to 6):** Look at next word



# The Top-Down Chart Parser (19/27)

 $\bullet$ 

 Example  $1. S \rightarrow NP VP$ 2. NP  $\rightarrow$  ART ADJ  $\,$  N 3. NP  $\rightarrow$  ART N 4. NP  $\rightarrow$  ADJ N 5. VP  $\rightarrow$  AUX VP 6. VP  $\rightarrow$  V NP  $\overline{ }$  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ the: ARTlarge: ADJ can: N, AUX hold: N, V Water: N **Loop 8**

**Enter ART2 (***"the"* **from 5 to 6):**





## The Top-Down Chart Parser (20/27)

 $\bullet$  Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 9**

**Enter N3 (***"water"* **from 6 to 7):**

Look at next word



## The Top-Down Chart Parser (21/27)

• Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 

**Loop 9**



the: ART large: ADJ can: N, AUX hold: N, V Water: N

**Enter N3 (***"water"* **from 6 to 7):**

( using the arc extension algorithm)



## The Top-Down Chart Parser (22/27)

 $\bullet$  Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



the: ART large: ADJ can: N, AUX hold: N, V Water: N

**Loop 10**

**Enter NP2 (***"the water"* **from 5 to 7):**



## The Top-Down Chart Parser (23/27)

• Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 10**



### The Top-Down Chart Parser (24/27)

• Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 11**

**Enter VP1 (***"holds the water"* **from 4 to 7):**



The Top-Down Chart Parser (25/27)



#### The Top-Down Chart Parser (26/27)

• Example  $_1$  The  $_2$  large  $_3$  can  $_4$  holds  $_5$  the  $_6$  water  $_7$ 



**Loop 12**

**Enter S1 (***"the large can holds the water"* **from 1 to 7):**



# The Top-Down Chart Parser (27/27)



Comparison between the Bottom-Up an Top-Down Chart Parsers

- The number of constituents generated by the top-down chart parser has dropped from 15 to 10
- In practice, the top-down method is considerably more efficient for any reasonable grammar